Problem **20-R-KM-DK-25**

In this problem, we have three arms arm AB arm BC, and CD, arm AB and BC, are free to move. And we're asked to find the angular velocity and angular acceleration of both of these arms. Meanwhile, arm CD is sliding along the bottom surface, we're given the velocity and acceleration of this arm. We're also given the dimensions and angles, angle locations. So let's start with a diagram. So what we're given is that CD moves on with a velocity of three meters per second to the right and an acceleration to the left. Now, since CD is a rigid body, we can assume that D and C will have the same acceleration and velocities and since it's sliding, there's no angular acceleration. So the velocities that are given we can simply apply them and velocity that acceleration, we can apply them at C. So this is going to be Vc. And it's going to be equal to three meters per second. And then in red, we're going to do acceleration. So acceleration is actually to the left so I'm going to offset it. But this is again acting at C, this is AC, and it's going to be equal to negative one meter per second square. And I'm going to draw in our coordinate system, our coordinate system is going to be centered at a but I'm going to leave it out of a, so I'm going to draw it to the side. So x is to the right, positive, y up positive and rotation is positive counterclockwise, as we'll see later. So essentially, we're given the velocity so we can scrap this sliding bar. And we can just assume that the system has two arms. And point C has the following velocity and acceleration. So we could solve for, use the formula V equals to omega cross r, to solve for the velocity at point B. And then again, omega at BC and AB. But in this case, we're actually going to use the time derivative technique to solve for the angular acceleration velocities of AB and BC. So what we first have to do is we have to define a distance. And this distance is all dependent on the problem. But it has to be a distance that we're going to call s, that is characteristic of your system, and starts from a point that has no velocity and goes to a point of known velocity and acceleration, so that when we take the derivative and the second derivative with respect to time of that distance, then we can directly plug in the linear velocity and acceleration that we're given. So in this problem, we're going to pick a distance between A and C. And this distance here will be called s. Okay, and so as you can see, this distance starts from a point with zero velocity and acceleration and goes to a point of known velocity and acceleration. And this distance changes with time as our whole system moves. And that's important. So now, we're going to write down this s in terms of the geometry of the problem. So in terms of the parameter L and theta, so as you can see, S is going to be equal to 0.5, which is r L times cosine theta. And this gives us the distance, this distance here. So this is 0.5 cosine theta. Because there's two of these, right, because this is symmetric, and so we're going to multiply it by two. And so as we can simplify to cos theta, and this is going to be the distance between point A and point C, at any time. Now we know that this changes with time and we know that at this specific instant, this S is changing with respect to with the same values VC, again, because this point is fixed. So it's this is not getting longer or shorter here, this is always going to be fixed here. But here, S is going to be expanding at three meters per second. And so we know that s dot the time derivative of s at this, in this frame, is going to be three meters per second. So if we take the time derivative of cosine theta, which is S, that we can equate to three meters per second and get a value for omega. So as you can see, when I take the derivative of s with respect to time, which is also denoted as S naught, I get the following. negative sine theta times theta dot. And if you're wondering where this data comes from, again, theta depends on time right because with time the state is going To increase or decrease. So when we take the derivative of cost data, we get negative sine theta, but then we also have to take the derivative for the with a product rule of the inside. So that's where this theta dot comes from, from the product rule. So now we have this equation, we can also rewrite theta.as omega, because the rate of change of the angle is the angular velocity. So we can rewrite this as sine theta times omega. And this omega, we're solving for the two different

Angular frequencies, right? But but as you can see, this problem is highly symmetric. So this angle is equal to this angle, and this length is equal to this length, therefore, the magnitude of these two Angular frequencies will be the same. And this is why we're solving for one omega, but it's important to note that their direction will be different. And we'll see how to get their direction in a little bit. But let's just solve this equation. So we know that s.is going to be equal to three. So we're just going to plug everything thing in three to the right, so positive is equal to negative sine of 60 degrees times omega. And if we solve for omega, we get that omega is equal to negative two square root of three radians per second. And again, this is not the final answer, this is just a magnitude now we need to find the direction. So as you can see, the direction depends on which way the angle or which way the rotation is occurring. So we defined a positive direction of rotation counterclockwise. So as you can see, angle theta of AB follows that convention, with an increase angle, we get rotation in the counterclockwise direction. Meanwhile, theta for armbc follows the opposite convention. So if we turn our, when we rotate our bar, counterclockwise, we should have an increasing angle, but our angle actually decreases. So whichever value we get our with our convention, so whichever value we solve for an omega, we actually have to reverse the direction. Okay, now let's find this direction. So we know that rotation is occurring in the xy plane. So the direction about which the rotation occurs is the Zed direction, which is out of the page. So in in case of arm AB, the angle is going to be decreasing because we got a negative, right, so m, omega is negative, the rate of change of the angle is decreasing. Therefore, the rotation, this change is going to be in the clockwise direction. And if it's clockwise, we see that it's opposite or sign convention, so it's going to be in the negative Zed direction, about the negatives that direction. So negative k hat direction with a unit vector. So omega A B, is going to be equal to negative two root three radians per second in the k hat direction. Meanwhile, for the other arm, you said, it's just the opposite. And we can think about it. So this arm is going to have a is going to actually so this arm is twisting Tor inwards, and this arm is twisting inwards, but with the opposite direction, so this direction is going to be clockwise. So since it's clockwise, it's going to have a positive rotation. Therefore, omega BC is going to be positive two root three radians per second. And again, it's the same k hat direction, because it's into, it's rotating about the Z plane, which is our into our out of the screen or the page. Alright, now let's move on to the angular acceleration. So angular acceleration we can find in a similar way, I mean, we could also find it with the formula A is equal to alpha cross r, but there's also that radio component. But we can find it with this time derivative method, where we take the double derivative of S or that their derivative with respect to time of s dot, and again, we have that value that is just one negative one meter per second squared. So let's go ahead and take that double derivative. So S double dot, I'm just going to be taking the derivative of this over here is simply the derivative with respect to time of negative sine theta times theta dot .so, This is going to involve the product rule. So there's going to be two terms that we get in the end. And so when we use the product rule, we get negative cosine theta theta dot squared minus sine theta times theta double dot. And again, this theta double dot here comes from the derivative of theta dot, where this theta dot squared comes from the product rule of the sine theta times again, this theta dot so that So we've got a square. And then again, we can do just like we did before plugging this value which we have, you can also plug in theta, which we have and theta dot we, just found over here. so we can simply plug that in. And we can solve for theta double dot, which is also alpha, our angular acceleration. So let's solve for that negative one, because it's in the negative direction is equal to negative cosine of 60 degrees, times two root three, squared. And notice that it doesn't matter what sign we put in here, because it's going to be squared. So whichever sign you put, you're going to end up canceling it out, we don't have to worry about that minus sign of 60 degrees, times alpha, which I just replaced for theta double dot, we can solve for alpha, just like we did with omega, we get that alpha is equal to 10 square root of three over three. And then we got a negative. But again, and this is radians per second squared. But again, this is doesn't have a direction, this is just a magnitude, we need to figure out the direction. So let's go back to our diagram before, since we can reason it with this direction here, AC is to the left. And on so the rate of change of velocity is going to be to the left, meaning that these two brackets are going to react in the opposite as it did for VC. Meaning that this bracket here will tend to move to that direction, and this will move in this direction, which is the opposite of what we had before. So the directions will actually be switched. So on alpha VC will be in the negative direction, and omega and alpha A B, you will be in the positive k hat direction. And so we can solve for it, we can simply write that down and box it in as your final answer. So out that a B is in the positive 10 square root three over three radians per second squared k hat direction. Meanwhile, Alpha BC is going to be in the negative 10 square root three over three radians per second squared k hat direction. And we can box that in as our final answer. So this is how you find the angular velocity and angular acceleration with the time derivative method of a simple geometry.