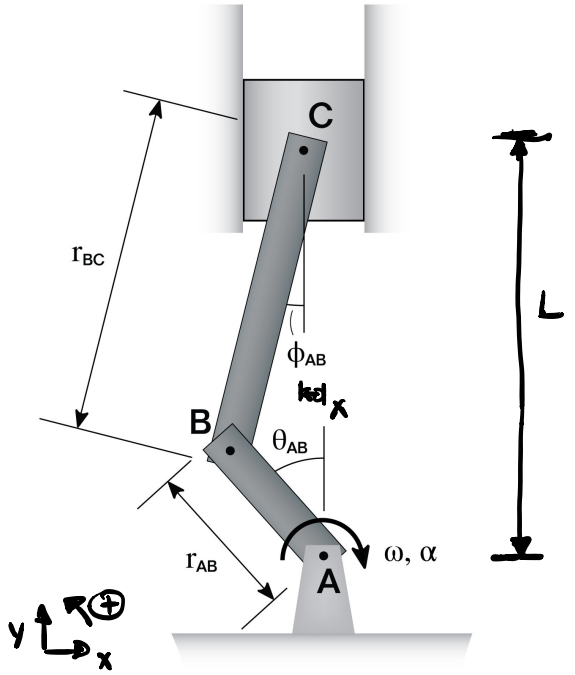
Students are attempting to create a lift to raise their model car. The lift is assembled with two linkages, link AB and link BC, as seen in the picture shown. If the links have length $l_{AB} = 0.2\ m$ and $l_{BC} = 0.4\ m$, determine the velocity and acceleration of the lift at the instant where the angular velocity of AB is $\omega_{AB} = -5\ rad/s$ and the angular acceleration of AB is $\alpha_{AB} = -7\ rad/s^2$.
Take the angles to be $\theta = 30\ deg$ and $\phi = 20\ deg$.



Solution:

$$L = 0.2\cos\theta + 0.4\cos\phi \quad [m]$$

$$\frac{dL}{dt} = -0.2\sin\theta\,\dot{\theta} - 0.4\sin\phi\,\dot{\phi} \quad [m/s] = V_L = V_C$$

$$\frac{d^2L}{dt^2} = -0.2\cos\theta\,\dot{\theta}^2 - 0.2\sin\theta\,\ddot{\theta} - 0.4\cos\phi\,\dot{\phi}^2 - 0.4\sin\phi\,\ddot{\phi} = a_L = a_C \quad [m/s^2]$$

$$x = 0.4\sin\phi - 0.2\sin\theta \quad [m]$$

$$V_x = \frac{dx}{dt} = 0.4\cos\phi\,\dot{\phi} - 0.2\cos\theta\,\dot{\theta} = 0 \quad [m/s]$$

$$\dot{\phi} = \frac{0.2 \cos\theta\, \dot{\theta}}{0.4 \cos\phi} = -2.304 \text{ rad/sec}$$

$$a_x = -0.4 \sin\phi\, \dot{\phi}^2 + 0.4 \cos\phi\, \ddot{\phi} + 0.2 \sin\theta\, \dot{\theta}^2 - 0.2 \cos\theta\, \ddot{\theta} = 0 \ [m/s^2]$$

$$\ddot{\phi} = \frac{0.4 \sin(20^\circ)(-2.304 \text{ rad/s})^2 - 0.2 \sin(30^\circ)(-5)^2 + 0.2 \cos(30^\circ)(-7)}{0.4 \cos(20)}$$

$$\ddot{\phi} = -7.945 \text{ rad/s}^2$$

$$V_c = (-0.2\text{m})\sin(30^\circ)(-5 \text{ rad/s}) - (0.4\text{m})(\sin(20^\circ))\left(\frac{0.2\text{m}\cos(30^\circ)(-5 \text{ rad/s})}{0.4\text{m}(\cos(20^\circ))}\right)$$

$$\boxed{V_c = 0.815 \text{ m/s}}$$

$$a_c = (-0.2\text{m})\cos(30^\circ)(-5 \text{ rad/s})^2 - 0.2\text{m}(\sin(30^\circ))(-7 \text{ rad/s}^2)$$
$$-(0.4\text{m})\cos(20^\circ)(-2.304 \text{ m/s})^2 - (0.4\text{m})\sin(20^\circ)(-7.945 \text{ m/s}^2)$$

$$\boxed{a_c = -4.54 \text{ m/s}^2}$$