Problem **20-R-KM-DK-26**

In this problem, we have a lift at C that can only move vertically attached to two arms arm a B and arm BC, we're given the angular acceleration and angular velocity of link A, B. And we're given the angles with respect to the vertical of both linkages, and a, b and link BC. And we're asked to find the velocity and acceleration of the lift. So point C. So again, I will draw in the coordinate system. So x will be positive to the right, y will be positive up and a positive rotation is counterclockwise. So and, again, because and this negative convention here will follow it being into the page. So we're gonna start with our solution. So there's two ways of solving this question. There's, we can use the method of finding the velocity, linear velocity and acceleration at point B, and then constraining the velocity of C to only be in the vertical direction, and then relating these two to find omega and alpha of link bc as well as the velocity and acceleration of C. But we can, in this question, we're going to use the time derivative method, which is essentially where we select length or a desired distance that changes in time, and we take the derivative, the first derivative to get the velocity second derivative to get the acceleration. And that essentially, is our acceleration and velocity for that point. So again, when we use the time derivative method, we always have to pick a distance between a point that is stationary, so that doesn't move and the point of interest. So the point that we're interested in the movement, and again, this distance, cannot change in direction, it has to keep a constant direction. So in this case, this distance is going to be between C and with an A, so A is the stationary point, and C is the point of interest. And so this here, as Ron in this diagram over here, will be our length of interest. And we'll call this L. Also L is the distance between point C and point A. So again, the rate of change of L is the same as the velocity of point C in the vertical direction, because there's only velocity in the vertical direction. Um, and so again, we have to find this distance L so that distance in terms of the variables of this geometrical system, so in terms of theta and the length of the two arms on so and we're given all of that information, so we can easily find that. So l is going to be equal to 0.2 cosine of theta plus 0.4 cosine of phi. And this is going to be again in meters. Okay, so I essentially just took the cosine of this length here, and the cosine of this length here to give us the full height. Okay, now I can take the first derivative with respect to time knowing that theta and phi change with time because everything is rotating, so we can take that derivative. So DL, dl over dt is going to be equal to negative 0.2 sine of theta and then we have a theta dot and that is because of the chain rule in there, because theta again changes with time minus zero point before sine of phi. Bye dot Okay, um, and all of this will be in meters per second. Now, we can take the second derivative, and that'll give us the acceleration. So this here is going to be equal to velocity of L which is equal to velocity of seat. Okay? You If we take d l, v squared L over dt squared, so the second derivative, essentially, we're deriving this equation here again, here, we're going to use the product rule, and again, also the chain rule. So we get the following. Negative 0.2 coasts, theta, theta dot squared minus 0.2 sine theta, theta double dot minus 0.4 coasts of phi phi dot squared minus 0.4 sine of phi phi double dot, this is going to be equal to A L which is equal to A of C, okay. And the units for this whole equation will be meters per second squared. Okay. So, again to reiterate, this here is the derivative of this. And there's two terms because we're doing the product rule. And then we also have the chain rule. And that's why we get the square term. And then again, these two terms are the derivative of this same exact method, just different angle and different value here. Now, we can essentially plug things in, so we have theta, we have theta dot, and we have theta double.we also have five but we're missing a few things. So we don't have phi dot, and we do not have phi double dot. And so we can't directly plug in and get values. But if you look at the system, the system is constrained right? See here can't move to the left or right. So in this case, we need to add that constraint to our calculations, because right now, these equations don't constrain, see for moving in the horizontal direction. Okay. So if we apply that constraint, then we'll have we can solve for phi dot and phi double dot and get those unknowns from this extra equations. So the way we constrain it is we say that the distance between C and A has to be set. So again, C and A are offset a little bit, right. And even if they weren't offset, if they were perfectly aligned, see the horizontal distance, the horizontal placement of C with respect to a should always be constant, so C here, and eight here, the distance between them should always be constant. Okay, so we're going to call this x, and x is essentially going to be this distance between the two. Okay, that is called x. And that distance needs to be constant, so the velocity needs to be zero. And again, the acceleration needs to be zero, so we're going to do the same exact process we did with L, we derive it and then instead of just having a value, we equate it to zero. And same thing with the acceleration, it's going to be zero. So let's do that same process for x. So x is going to be equal to instead of cosines. Here, we're going to have signs. And again, because we're going to subtract the two, so we're going to take this component here and subtract this whole thing to get this little distance, subtract this distance here. So let's do that. We're going to have 0.4 sine of phi minus 0.2 sine of theta. And again, this is all in meters. So again, to reiterate, this distance x is this distance here, the offset distance between the two. And that has to be constant. So essentially, what I did is the first sign comes from the whole length over here. And I subtract this the length from this point to here, and the subtraction leads me to this point, because this distance, this thing here minus this whole thing, minus this whole thing over here gives me this little distance. So if we take the derivative and equate it to zero, we can solve for phi dot. Okay, so we have VX is equal to dx over dt and if we take this derivative we get 0.4 Coase of phi phi dot minus 0.2, close beta theta dot, okay. And again, this is going to be equal to zero, right, and all the times in this equation are meters per second. Now we can since we equated to zero, we have phi, we have theta, we have theta.we can solve for y dot. And so if we solve for it, we get that I'm not going to plug in the values manually, but we get that phi.is equal to 0.2 cosine of theta theta dot over 0.4 coasts of phi, which is equal to negative 2.304 radians per second. Okay, so now we have sulfur phi.we. Now, what remains is a phi double dot over here and then we're set to plug everything into these equations. Okay. So, the way we solve for it is again, the velocity of x needs to be zero. So this distance here needs to be set. So that this point C doesn't travel in the X or Y direction. Now we also have to set its acceleration to zero, so we take the derivative of this or the double derivative of that. So again, a x is going to be equal to negative 0.4 sine phi times phi dot squared plus 0.4 coasts of phi times phi double dot plus 0.2 sine of theta theta dot squared minus 0.2 cosine of theta theta double dot. And again, these derivatives were taken in the same method that we did before. And now we can equate this to zero. And again, everything in this equation is in meters per second squared. When we equate this to zero, we are going to, we can solve for this phi double dot, because we have phi dot from what we just got, we have fi we have theta and we have theta.we have theta double.so, we can plug everything in and get the following 0.4 sine of 20 degrees times negative 2.304 radians per second. And this is going to be squared minus 0.2 sine of 30 degrees times negative five squared plus 0.2 coasts of 30 degrees, negative seven, all divided by 0.4 cosine of 20. And essentially, you can solve for this and we get that by double.is equal to negative 7.945 radians per second squared. Okay, and now we have all of our unknowns. So again, this here is high. And we can plug them into these equations to solve for VC and AC, we just plug all of those in. And so if we do that, we get the following. vc is going to be equal to negative 0.2 meters times sine of 30 degrees times negative five and this radians per second Minus 0.4 meters times sine of 20 degrees times 0.2 meters coasts of 30 degrees times negative five radians per second divided by 0.4 meters times cos of 20 degrees. And so VC is going to be equal to 0.815 meters per second. And we have seen how we can solve for AC. And AC is going to be equal to if we plug everything in notice 0.2 meters as cosine of 30 degrees times negative five radians per second to the power of two minus 0.2 meters times sine of 30 degrees times negative seven radians per second squared. And then we have, I'm going to go on the next line minus 0.4 meters cos of 20 degrees turns negative 2.304 meters per second to the power of two minus 0.4 meters. Sine of 20 degrees times negative 7.9 or four. Alright, sorry, meters per second squared. If we solve for this, we get that AC is equal to negative 4.54 meters per second squared. So that's our final answers. VC and AC