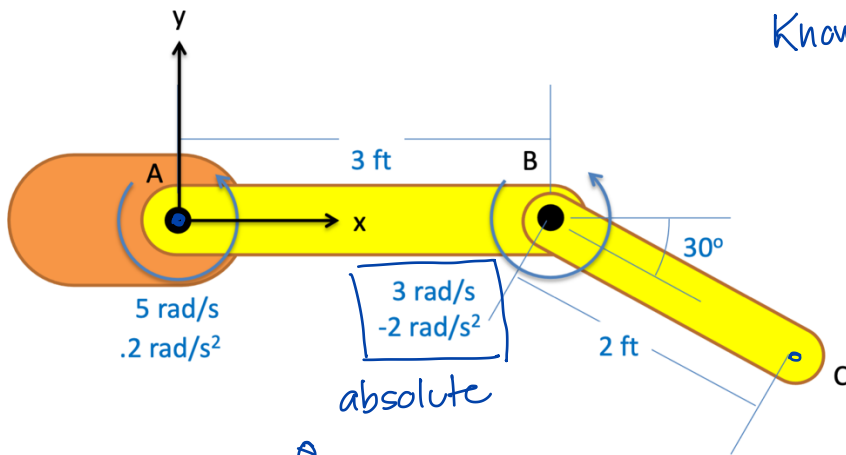


The robotic arm shown below has a fixed orange base at A and fixed length members AB and BC. Motors at A and B allow for rotational motion at the joints. Based on the angular velocities and accelerations shown at each joint (described with respect to the fixed frame), determine the velocity and the acceleration of the end effector at C.



Find  $\vec{v}_c$  ;  $\vec{a}_c$

Known:  $\vec{v}_A = 0$

$\vec{a}_A = 0$

$\vec{\omega}_{AB} = 5 \text{ rad/s } \hat{k}$

$\vec{\alpha}_{AB} = 0.2 \text{ rad/s}^2 \hat{k}$

$\vec{\omega}_{BC} = 3 \text{ rad/s } \hat{k}$

$\vec{\alpha}_{BC} = -2 \text{ rad/s}^2 \hat{k}$

$\vec{r}_{B/A} = 3 \text{ ft } \hat{i}$

$\vec{r}_{C/B} = 2(\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j}) \text{ ft}$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$\vec{v}_C = \vec{v}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$$

$$= \vec{\omega}_{AB} \times \vec{r}_{B/A} + \vec{\omega}_{BC} \times \vec{r}_{C/B}$$

$$= 5 \text{ rad/s } \hat{k} \times 3 \text{ ft } \hat{i} + 3 \text{ rad/s } \hat{k} \times (2 \cos 30^\circ \hat{i} - 2 \sin 30^\circ \hat{j}) \text{ ft}$$

$$= 15 \text{ ft/s } \hat{j} + 6 \cos 30^\circ \text{ ft/s } \hat{j} + 6 \sin 30^\circ \text{ ft/s } \hat{i}$$

$$\boxed{\vec{v}_c = (3 \hat{i} + 20.2 \hat{j}) \text{ ft/s}}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$

$$\vec{a}_c = \vec{a}_B + \vec{\alpha}_{BC} \times \vec{r}_{C/B} - \omega_{BC}^2 \vec{r}_{C/B}$$

$$= \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A} + \vec{\alpha}_{BC} \times \vec{r}_{C/B} - \omega_{BC}^2 \vec{r}_{C/B}$$

$$= 0.2 \text{ rad/s}^2 \hat{k} \times 3 \text{ ft } \hat{i} - (5 \text{ rad/s})^2 3 \text{ ft } \hat{i}$$

$$- 2 \text{ rad/s}^2 \hat{k} \times (2 \cos 30^\circ \hat{i} - 2 \sin 30^\circ \hat{j}) - (3 \text{ rad/s})^2 (2 \cos 30^\circ \hat{i} - 2 \sin 30^\circ \hat{j}) \text{ ft}$$

$$= 0.6 \text{ ft/s}^2 \hat{j} - 75 \text{ ft/s}^2 \hat{i} - 4 \cos 30^\circ \text{ ft/s}^2 \hat{j} - 4 \sin 30^\circ \text{ ft/s}^2 \hat{i}$$

$$- 18 \cos 30^\circ \text{ ft/s}^2 \hat{i} + 18 \sin 30^\circ \text{ ft/s}^2 \hat{j}$$

$$\boxed{\vec{a}_c = (-92.6 \hat{i} + 6.1 \hat{j}) \text{ ft/s}^2}$$