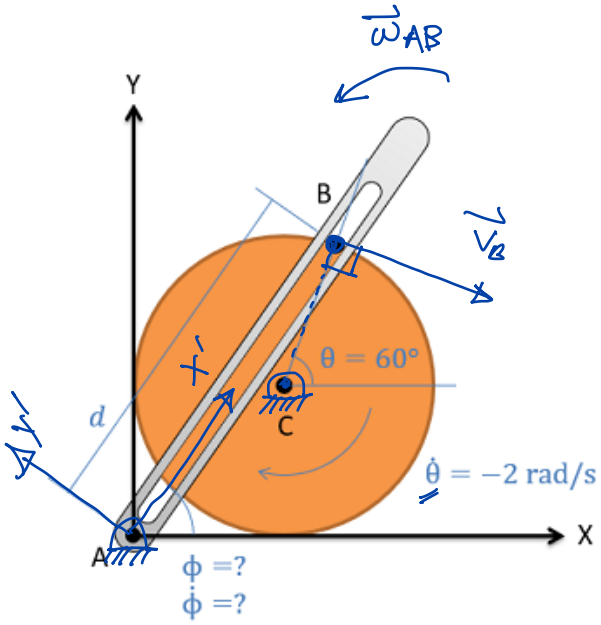
The crank-rocker mechanism as shown below consists of a crank with a radius of .5 meters rotating about its fixed center at C at a constant rate of 2 rad/s clockwise. Rocker AB fixed at its base at A and connects to point B along the edge of the crank. The pin at point B can slide along a frictionless slot in AB. In the current state, what is the angular velocity of rocker AB?



Find $\vec{\omega}_{AB}$

Known:

$\vec{V}_A = 0$

$\vec{V}_C = 0$

$\vec{\omega}_{BC} = -2 \text{ rad/s}\,\hat{k}$

$\vec{r}_{B/C} = 0.5(\cos 60\,\hat{i} + \sin 60\,\hat{j})\,m$
$= (0.25\,\hat{i} + 0.433\,\hat{j})\,m$

$\vec{r}_{B/A} = d(\cos\phi\,\hat{i} + \sin\phi\,\hat{j})\,m$

$\vec{r}_{C/A} = (0.5\,\hat{i} + 0.5\,\hat{j})\,m$

Assume: $\vec{\omega}_{AB} = \omega_{AB}\,\hat{k}$

Geometry:

$\vec{r}_{B/A} = \vec{r}_{C/A} + \vec{r}_{B/C}$

$d\cos\phi\,\hat{i} + d\sin\phi\,\hat{j} = 0.5\,\hat{i} + 0.5\,\hat{j} + 0.25\,\hat{i} + 0.433\,\hat{j}$

$\hat{i}$: $d\cos\phi = 0.75$ $\quad\Rightarrow\quad d = \dfrac{0.75}{\cos\phi}$

$\hat{j}$: $d\sin\phi = 0.933$ $\quad\Rightarrow\quad 0.75\dfrac{\sin\phi}{\cos\phi} = 0.933$, $\tan\phi = 1.244$

$\Rightarrow \phi = 51.2°$, $d = 1.20\,m$

Two equations for $\vec{V}_B$:

$\vec{V}_B = \vec{V}_C^{\,0} + \vec{\omega}_{BC} \times \vec{r}_{B/C}$

trans. frames

$= -2 \text{ rad/s}\,\hat{k} \times (0.25\,\hat{i} + 0.433\,\hat{j})\,m$

$= (0.866\,\hat{i} - 0.5\,\hat{j})\,m/s$

rot frames: $\vec{v}_B = \cancel{\vec{v}_A}^{\,0} + \vec{\omega}_{AB} \times \vec{r}_{B/A} + (\vec{v}_{B/A})_{rel}$

$= \omega_{AB}\hat{k}' \times 1.2\text{m}\,\hat{i}' + (v_{B/A})_{rel}\,\hat{i}'$

$= 1.2\,\omega_{AB}\,\hat{j}' + (v_{B/A})_{rel}\,\hat{i}'$

$\phi = 51.2^\circ$

write $\vec{v}_B$ from trans frames in $x'y'z'$: $\vec{v}_B = 0.866\hat{i} - 0.5\hat{j}$

$= 0.866(\cos\phi\,\hat{i}' - \sin\phi\,\hat{j}') - 0.5(\cos\phi\,\hat{j}' + \sin\phi\,\hat{i}')$

$= 0.153\hat{i}' - 0.988\hat{j}'$

$0.866^2 + 0.5^2 = 0.153^2 + 0.988^2$ ✓

equate: $0.153\hat{i}' - 0.988\hat{j}' = 1.2\,\omega_{AB}\hat{j}' + (v_{B/A})_{rel}\,\hat{i}'$

components:

$\hat{i}'$: $0.153 = (v_{B/A})_{rel}$

$\hat{j}'$: $-0.988 = 1.2\,\omega_{AB} \Rightarrow \omega_{AB} = -0.824\,\text{rad/s}$

$$\boxed{\vec{\omega}_{AB} = -0.824\,\text{rad/s}\,\hat{k}}$$