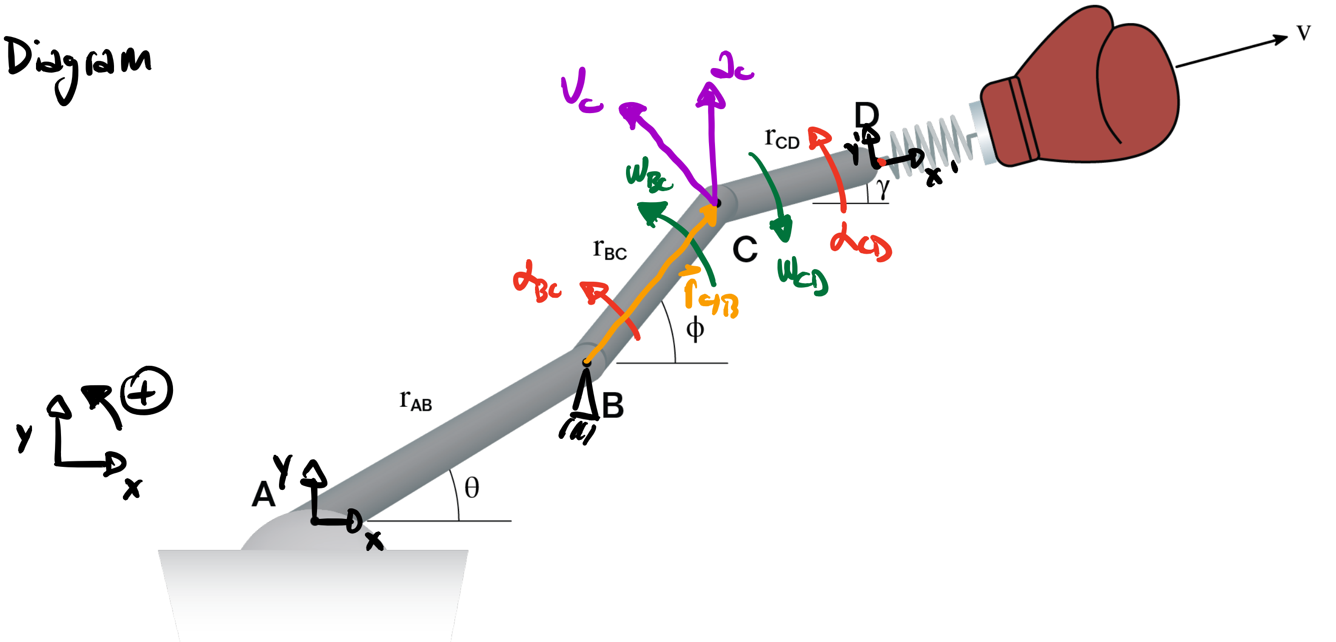
A very funny engineer has attached her prank punching machine to a robotic arm. If the boxing glove moves at a maximum 1m/s constant velocity, relative to the link CD, what is the velocity and acceleration of the glove at the instant shown?
The lengths of the linkage arms are given as $r_{AB} = 0.3\ m$, $r_{BC} = 0.6\ m$ and $r_{CD} = 0.4\ m$. The angles are given as $\theta = 30\ deg$, $\phi = 50\ deg$ and $\gamma = 15\ deg$. Arm BC is rotating at $\omega_{BC} = 2\ rad/s$ and $\alpha_{BC} = 1\ rad/s^2$, while arm CD is rotating at $\omega_{CD} = -0.25\ rad/s$ and $\alpha_{CD} = 0.25\ rad/s^2$. Arm AB is stationary throughout.

① Diagram



Solution

$$\vec{v}_C = \vec{\omega}_{BC} \times \vec{r}_{C/B} = 2\hat{k} \times (0.6\cos 50^\circ\,\hat{i} + 0.6\sin 50^\circ\,\hat{j})$$

$$\vec{v}_C = 1.2\,(\cos 50^\circ\,\hat{j} - \sin 50^\circ\,\hat{i})\ m/s$$

$$\vec{a}_C = \cancel{\vec{a}_B}^{\,0} + \vec{\alpha}_{BC} \times \vec{r}_{C/B} - \omega_{BC}^2\,\vec{r}_{C/B}$$

$$= 1\hat{k} \times (0.6\cos 50^\circ\,\hat{i} + 0.6\sin 50^\circ\,\hat{j}) - 2^2(0.6\cos 50^\circ\,\hat{i} + 0.6\sin 50^\circ\,\hat{j})$$

$$= 0.6\cos 50\,\hat{j} - 0.6\sin 50^\circ\,\hat{i} - 2.4\cos 50\,\hat{i} - 2.4\sin 50^\circ\,\hat{j}$$

$$\vec{v}_D = \vec{v}_C + \vec{\omega}_{CD} \times \vec{r}_{D/C} + (\vec{v}_{D/C})_{x'y'z'}$$

$$= 1.2\cos 50^\circ\,\hat{j} - 1.2\sin 50^\circ\,\hat{i} + (-0.25\,\hat{k}) \times (0.4\cos 15^\circ\,\hat{i} + 0.4\sin 15^\circ\,\hat{j}) + (\cos 15^\circ\,\hat{i} + \sin 15^\circ\,\hat{j})$$

$$\boxed{\vec{v}_D = 0.0726\,\hat{i} + 0.934\,\hat{j}\ m/s}$$

$$\vec{a}_D = \vec{a}_C + \vec{\alpha}_{CD} \times \vec{r}_{D/C} - \omega_{CD}^2 \times \vec{r}_{D/C} + 2\vec{\omega}_{CD} \times (\vec{v}_{C/D})_{x'y'z'} + \cancel{(\vec{a}_{D/C})}_{x'y'z'}^{0}$$

$$= -0.6\sin 50^\circ \hat{i} - 2.4\cos 50^\circ \hat{i} - 2.4\sin 50^\circ \hat{j} + 0.6\cos 50^\circ \hat{j}$$

$$+(0.25\hat{k}) \times (0.4\cos 15^\circ \hat{i} + 0.4\sin 15^\circ \hat{j}) + 2(-0.25\hat{k}) \times (\cos 15^\circ \hat{i} + \sin 15^\circ \hat{j})$$

$$-(0.25)^2 (0.4\cos 15^\circ \hat{i} + 0.4\sin 15^\circ \hat{j}) + 0$$

$$\boxed{\vec{a}_D = -1.92\,\hat{i} - 1.85\,\hat{j} \quad m/s^2}$$