Problem **20-R-KIN-DK-18**

In this question, you're hauling a heavy cart up a slope of 30 degrees, you're pulling it with a force of 500 Newtons, at an angle of 42 degrees with respect to the horizontal, the mass of the cart is 30 kilograms. And all of the dimensions of the wheels and the center of gravity and the load load application zone are given. And you're asked to find the normal forces on the wheel. So we'll a and we will be along with the acceleration of the cart up the slope. So looking at the diagram here, we're going to first draw in the forces, and then we're going to draw a freebody diagram. So we're gonna definitely have normal forces and the normal forces are perpendicular to the surface at the bottom. So this we're going to call NB, and so we're going to call normal force at A, then we're going to have a force due to gravity, which points down Fg. And again, we have this force here, F, which is applied at that location over there. And then last thing we have is to draw our kinetic kinetic diagram, where we have an acceleration. And this is going to be along the slope, because we're assuming that the cart will not detach. So it's just going to be along the slope direction. Alright, so since we drew all their forces, now we can draw in our coordinate system. And for simplicity, you always want to pick a coordinate system that reduces the amount of calculations, especially when you're doing a moment balance. So in this case, this force here, this force here, and this force here are all perpendicular or parallel to the slope direction. So we want to pick the coordinate system to be aligned with that. So we don't have to take so many cosines and sines, because we're three forces are just going to be perpendicular or parallel. Whereas two are going to need to involve cosine and sine. And since we're given the angles for these, much simpler to calculate the components along the x and y direction, for these two forces, then for three, and then acceleration is not a force. But again, acceleration will come in later into our force balance, so its direction matters. So we're going to pick a coordinate system to be along the slope. So y will be up that way, x will be positive this way. So this is x, this is y, and we're going to assume a positive rotation to be counterclockwise with respect to that x and y. So again, let's draw our freebody diagram. So remember, with a freebody diagram, you have to detach the body. From the surfaces, it's connected to. inclined, then we have m g, which is equal to f, g. And if we draw this, this angle here is going to be 30 degrees, then we have our force F here, which is slanted up at an angle, and this angle here is going to be phi. Whereas this here is equal to theta. Then we have the two wheel forces, and remember that this here is going to be to NB. And this year is going to be a two an A, and we have a two there, because there's two wheels, one on the front, the one that we see and one behind it. And since they're aligned, they're going to have the same force because they're in the same location. But again, there's two of them. So whichever force you get, if you calculate it for one wheel, then you'd have to divide it by two to get the two wheels. And then we have to draw in our acceleration, which we are assuming to be, again acting at the center of gravity, AG, but we're assuming that the direction is going to be parallel to or in the x direction, so parallel to the slope. And that is because we're assuming that this car is not detaching from the garden ground, so it's not going to start to rotate. So alpha is going to be zero and the acceleration can only be along that because it's not going to lift off. Hey, so this is our full freebody diagram. Let me draw in the coordinate system for reference one more time. So this is y this is x and this is a positive rotation. Okay, and this is b, this is point A, this is point G, just for labeling sake. All right, let's do our force balance. So we're going to start with our x force balance in the x direction, then in the y direction, and then we're going to do a moment balance. And that should give us enough equations to solve for the three unknowns. So the three unknowns are the acceleration, and a and b. Okay, so let's start with some forces in the x direction. And this is going to be equal to the mass times the acceleration of G. Again, the full acceleration here, because we're assuming acceleration is only in the negative x direction, so up the slope. So let's do this force balance here, we have negative f coasts, phi, plus F, G, sine theta is equal to A G. Hey, so in the x direction, we only have the two components from these forces, these are only purely in the y direction, so they don't add up, they equal to A G. So in this case, we can actually solve for ag directly, because we have F and we have f g, and we have the angles. so we can directly plug in numbers and solve for ag. So a G is equal to negative 500 Newton's coasts of 42 degrees, plus 30 kilograms, times 9.81 meters per second squared, times sine of 30 degrees. And this is all divided by the mass, which is 30 kilograms. And this yields ag is equal to negative 7.48 meters per second squared. And so this is our first answer, acceleration at G. And just for a sanity check, we see that this is negative, which is expected because this block should be traveling up the slope. So the sign is correct. All right, now let's do our sum of forces in the y direction, sum of forces in y is equal to zero and again, zero here, because there is no acceleration in the y direction. So this is nice and simple, f sine phi minus f g coasts, theta, plus two and B plus to an eight, a is equal to zero. And we can see that in this equation, we have both NB and an A, so we can't directly solve so we're gonna have to get another equation and then solve the system of two equations, two unknowns. So next up, we have our moment balance. So for a moment balance, we have to pick a location about which we do our moment balance. So in this case, here, if you want to simplify your life, you always want to take the moment at a location where you know, the force or where you don't know the force. So wherever you have an unknown, because if for example, we take their sum of moments at a here, this force na doesn't create a moment about a, right because it just crosses through. So then in our equation, we won't have this unknown. And our equation will be a function of all the other unknowns, and in this case, we know ag from our first equation. so we can directly solve for to NB without solving a system of two equations, two unknowns. So again, it's really important that you pick you can pick any location to take the moment balance about wish, but easier, usually, you either take it or the location where you have an unknown, or at a location where you have many forces that are slanted at different angles. So you can cancel out those complex cross products and simplify it to just other unknowns. Okay, so really important to take a moment about the correct location to simplify your life. So here, we're gonna take it about a, we could have done B but I just picked a for simplicity, and a G times d. I remember when you're taking the moment about a location that is not the center of gravity, you are going to have this acceleration term. Okay? So this acceleration at the center of gravity will also create a moment about a about the point that we're taking that is not the center of gravity. If this was the center of gravity, the distance here would be zero. So this is again, this the moment are between the location A, and the force or the acceleration ag. So, this only applies if you are taking the moment about a point that is not the center of gravity, if I'd taken the point about moment about the center of gravity, which is still worked, but it wouldn't have had that term, because it would have been zero. Anyways, back to doing our moment balance, we are going to take it for us one by one and do a full moment balance about eight. So let's start with fg, so negative fg, sine theta times h. So this is going to be fg here, h is this distance here. So we're taking this component here. So that's why we have the sign. Okay. So again, would be in this direction, this is the moment arm, this is h, and that's fg sine theta. And it's negative because it's twisting everything in the opposite direction as our positive direction. Next, we have the other component of f g plus f, g coasts, theta times dA. Okay, so this is the other components. So this is the vertical component, the y component of f, g. And so this distance here is d a, and that's the moment arm and this is the cosine of that force of fg, cosine of 30 degrees. Next step, we're going to move on to the other forces. So I'm going to go on a new line plus f coasts of phi times y. So again, why is the distance so coasts, f coasts is this component over here? And why is that distance over there, okay, which is different than this distance over here, which is H, okay. Then we have the other component of F, which is negative f sine phi times the three distances, so we need this distance here now. So it's going to be the distance of DA DB plus x v. So da plus DB plus x, b, then we have the normal force at B. So we have minus two and B times d A plus d, b. Again, the normal force that B acts on this direction, so we need this moment arm here, which is simply d A plus dB. And then we're going to equate all of this to M, a Y, or a G, which is in the y direction, times h, because again, h is this distance over here, and since a gx in that direction over there. We need this moment arm, which is simply each. So now we have an equation. And this equation has one unknown, which is simply NB. Because we know ag from the previous equation there, we can simply plug everything in and solve for NB, which is what I'm going to do. So I'm going to do it over here. I'm going to plug in the numbers on negative 30 kilograms, times 9.81 meters per second squared times sine of 30 degrees times 0.5 meters, which is H plus 30 kilograms, times 9.81 meters per second squared times cosine of 30 degrees times 0.3 meters, which is da. Okay, new line plus 500 Newton's which is F times coasts of 42 degrees, r times 0.4 meters minus 500 Newtons times sine of 42 degrees times 0.6 meters. And again, the 0.6 is adding all of those distance As up minus two and B times 0.55 meters in this 0.55 meters is da plus dB. And this is going to be equal to 30 kilograms, which is the mass times negative. And here we're going to add our acceleration 7.48 meters per second squared times 0.5 meters, which again is H. Okay, now we plugged everything in, we can simplify, we can move all of this stuff to the right hand side and divide by negative two and divide by 0.55 and solve for NB. So and B is going to be equal to 57.26 Newtons, okay. And then we have NB, so we can go back to our for some forces in the y direction, we can plug in and B we know all of these unknowns, and we can simplify simply solve for na. And so when we do that, we get that NA is equal to a 97.11 Newton's. And so these are our final answers for the normal forces of the wheels. And remember, this is the normal force on each wheel. There's four wheel so NB is the normal force on the front wheels. And NA is on the back wheels.