Problem **20-R-KIN-DK-41**

In this problem, we're towing a cart with a block on top of it. And the cart has a mass and the block has a mass and a uniform density. And we are asked to find what is the maximum force we can apply to this cart before the whole block starts to tip. Okay, so we're going to start with our free body diagram, as always, and in this case, we're going to have two free body diagrams, one for the cart, and one for the block. Again, because we have to analyze these separately. So we're going to start with the cart free body diagram. So our cart, and so cart is going to include the block on top of it. So this is our block, and everything is attached together, and then we have our two wheels over here. So the forces on this free body diagram are as follows. We have a force that is pulling the cart in that direction, we have a force due to gravity, fg of a that's the force of gravity due to the cart. And then we have a force of gravity due to the luggage. And then we have a force of gravity due to the cart, g of the cart. And then we have two normal forces. And one and m two. Okay. And then lastly, we have an acceleration of the cart and the block, and we're going to call this a. And, again, we're going to specify a coordinate system. So our coordinate system is positive x to the right, positive y, and moments are positive counterclockwise. Okay. So this is our free body diagram of the whole system together cart and block. And you'll see why we need that we need this later. And then we can do a second free body diagram of just the block. So the block is going to have again, the same force due to gravity FgA. And it's going to have a normal for us along with a friction for us on the bottom. Okay, now, it's really important that we determine the location of this normal and friction force. So in the question, we're given that we need to find the force here, right before this whole block here starts tipping. Okay, so if everything was resting, I'm going to draw this in green. If there was no force F, and we'd have a gravitational force, FgA, that balance that is balanced by a normal force. And right, and this normal force would be right aligned with the center here. But as this block accelerates, the normal force actually starts moving backwards. So if the acceleration is in that direction, A, then we get that this normal force here, we'll start moving backwards in that direction. and it moves further and further back until we get to this point here. And this is the right at the edge of the block. Okay, once the normal force moves to that edge there, that's the point where if it moves further, the whole block is going to start to rotate, so it's going to create an omega and an alpha in that direction. Okay, and the whole block is going to start tipping backwards. So again, to impose this condition of tipping, so just before tipping, we need to apply the normal force right at the edge of that block. Okay, and again, with a normal force comes a friction force. And so the friction force, I'll draw this on in purple will be in this direction. And there's always a friction force. Again, countering this acceleration here. But in this problem, we're not given any information about the friction coefficient, we're just given that it's going to start tipping before sliding. Okay, but you'll see why we don't need the friction coefficient. But back to our free body diagram, we're going to add our force, or normal force right at the edge of this block. And so this is going to be on the normal force a, and then we have our friction force f of f of a. Okay. And then lastly, we're going to, we're going to add our kinetic diagram portion, which is an acceleration at the center of gravity, a. And this is the same acceleration as the whole cart, right? Because there's no relative motion between the two, there's no sliding, they're going to have the same acceleration. Okay, so these are two freebody diagrams. And we're going to add some naming conventions. So this point here, or we're going to call Oh, and you'll see why we need that later. Okay. And this is block a, and this is the cart. This is a, an O is right there. Okay, now that we have our free body diagrams, you can start with our force balances. So in this case, what we're trying to solve is for the force F, okay, and our unknowns are, well, the two normal forces, and acceleration along with F, okay, but you'll see why we don't need to solve for all of these, and then also the friction force and the normal force, we don't need to solve for these in detail, we just need to solve for a few equations that really simplify your problem. Okay. So let's start with the cart. So in the cart, again, we can do all three equations, but we can intuitively think and reason why we don't need to take all the sum of forces. Okay, so if we take the sum of forces in X, we're going to relate F with a. And we need that we need to relate F with a. Okay, so that's why we are going to take a sum of forces a sum of force and x for the cart. So let's do that over here. Some forces in the x direction is equal to ma. And let's implement this. So sum of forces in the x direction, we just have F, an F is going to be in the negative x direction, because we define this to be x, this to be y in this to be a positive rotation. So we're going to have negative F is equal to m, and m here is the whole mass of the whole system. Okay? Again, in this free body diagram, we only include external forces. So this these internal forces, they're canceled because they're opposite in their opposite in magnitude in direction and same in magnitude on the cart compared to the block. So they cancel out. This is only external forces, but we have two masses here. Okay. So we have to add both the mass of the cart and the mass of the block. So m of A plus m of the cart. And this is going to be times the acceleration a, which we said was just going to be an x because it's right before it started stripping, so no vertical accelerations. Okay. So when we solve for this, we try and solve for this, but we see that we have two unknowns, right? f and a, we don't know them. So this equation is useful because it relates F and a, but we can't solve for anything yet. Now let's try and do a force balance in the y direction. And we can see that this doesn't yield us any useful information. Because if we sum up the forces in Y, there's these four forces 1234. And we're adding two unknowns that we don't need to solve for. And it's not relating any of the other unknowns that we actually need to solve for. So if someone forces in y does not yield any useful information for us, it would if we have to solve for n one and n two, okay? And then if we do a sum of torques on again, this doesn't yield any information. And we also cannot do this because we're not given any of these distances here, between the wheels. So since we're not given the geometry, that there's no information to relate those forces, okay, so sum of moments for the cart is not useful for us. So we conclude that for the cart, the only useful equation is the sum of forces in the x direction. So now we need more equations. That's why we're going to look at the block. Now if we look at the block Again, we can do a sum of forces in the x direction. Okay, so let me draw in the x direction is this way, y direction is that way and rotation is positive counterclockwise. If we do a sum of forces in the x direction, we're going to relate the force of friction with acceleration solely. Okay. And since we don't need to solve for the force of friction, we don't need this equation. Let's do a sum of forces in the Y. If we do that, we are going to really the gravitational force on the block to the normal force. And since we're not solving for normal for us at a, we also do not need this equation. So the last one is the sum of moments. And if we do a sum of moments, we are going to pick a point we're going to relate this force with the acceleration, which is what we need. So we have two equations relating force and acceleration, which gives us the which we can solve two equations, two unknowns. So let's do that. So for the block, all we're going to do a sum of moments. And it's really important that we pick the right location to do this on moments about because if we pick a random point, see here, we would not get the number of equations and unknowns that we're going to, we can solve the problem with, okay? If we pick Oh though, that's going to cancel our normal force and our force of friction because they go straight through that point, no moment arm, no torque, okay. And so if we pick Oh, we're just having an equation relating A and F, G, A, which is what we need, okay. And so that's why we need, we need to take moments about oh, this is a smart choice, you could also take a moment about anywhere else around the block, but then you would have to take the summer force in the x and a summer force in the y to relate everything and get back to the original to the problem. And you would solve four equations, four unknowns instead of two equations, two unknowns. And again, this sum of moments here is going to be equal to m A times d. Okay, and because remember, when we take the sum of moments by about a point, that is not the center of mass center of gravity of the block, we need to include this term that arises from the acceleration, okay, and this is the moment arm that the acceleration makes with the points that we're looking to do a sum of moments about. So we can do this, and this is what we get m g, times with a divided by two. And this is because this moment arm is going to be half of this distance, and g is F ga just rewritten is equal to a times h A over two, again, this distance d here is going to be half of the height. Okay? So that's why we have h A over two. Okay. And this is a simple equation that we can directly see where we can directly solve for A, the acceleration. And again, this here, the way we implemented the tipping condition is by applying these normal forces and for two horses right at the edge of the block. Okay, so we're not implementing any other conditions, because by applying those forces there, we're assuming that we have, we're just before tipping, okay, so we can plug values in. So we have 15 kilograms, times 9.81 meters per second squared, times 0.5 meters over two is going to be equal to 50 kilograms times A times 0.8 meters over two. Okay? And if we solve for A, you get that A is equal to 6.131 meters per second squared. This is not our final answer, though. We're not asked to find a we're actually asked to find F. But since we have a, then we can plug it into this equation to solve for F. Okay, so we go back to the sum of forces and we plug in a, so we have 50 kilograms plus 12 kilograms times six Point 131 meters per second squared is going to be equal to 380.14 Newtons and this is going to be equal to our force. Okay, and we get a negative because it's in the negative x direction. So f is equal to negative 380.14 Newtons in the i hat direction. And this is our final answer.