Problem **20-R-KIN-DK-25**

In this problem, we have this pendulum. So playground ride for which we have to determine the length, that yields a maximum angular acceleration of five radians per second squared when no one is sitting on it. So we have a rod of length l that weighs 0.6 kilograms, no matter which length. And at the bottom of it, there's a disc of radius 0.3 meters with mass with one kilogram attached at the end of it. And we have to analyze the instant of which angle theta is 45 degrees. So we have to determine this length that leads to this maximum angular acceleration. So we know that a longer length leads to larger Angular accelerations. So our length will be the maximum length. Okay. So let's start with the freebody diagram. So we know I'm going to draw it onto this image here, and then I'm going to draw it separately. So we know there's a force of gravity due to gravity are both the center of the rod and the center of the disk at the bottom, this is going to be f g, and then we have F capital G here. So this is the disk and this here is the rod. Okay. And then we're going to have reaction forces at A. But the most important thing is drawing in alpha, which is already drawn in this diagram as the angular acceleration. So we have x in that direction, y is positive in that direction, and rotation is positive counterclockwise. Let's draw a separate freebody diagram, because when we draw a freebody diagram, we want to detach the system. So let me draw it in to the bottom there. It's not exactly a disk, but that'll do. So let me draw in the forces. So we have here at point A, we have reaction forces in X, and Y, and in x, we have ax. And then we have the two gravitational forces. So FG, and FG, these act at the centers of gravity, of the rod and the disk. And then we have our coordinate system, which is x positive this way, why positive this way with a positive rotation. And lastly, we're going to have our alpha, angular acceleration. Okay. So now we can we have a freebody diagram. So we can do a sum of forces and moments but and then Oh, I forgot to mention this angle here is 45 degrees. So this angle here is 45 degrees. So back to the sum of forces and moments, if we take the sum of force in the x direction, y direction, all we are going to solve for is these components ax and ay, the reaction components. And we don't really need to solve for that we're not asked for the reaction, we're only asked for the length of this rod, which doesn't depend on the reaction. So we don't really need the sum of forces in x&y, what we need is the sum of moments because the sum of moments relates this alpha to the geometrical properties of length, l, and through these forces that create a moment. And so it's important that we take a sum of moments about a, so that we cancel out these two forces that are unknown. And all we have is these known forces that are in their moment is going to depend on this length L here. So that's why how we relate l to alpha. So let's do that. So we take a sum of moments about point A, and we're going to equate it to I alpha. So, let's, I'm going to solve for AI a later I'm just going to leave it but essentially we have ay ay ay ay. so this is an AI about a which is different than about the center of gravity. And then we multiplied by alpha And since there's no movement at alpha, then we're going to have this equal to the sum of moments. And there's two forces that create moments F, lowercase G, and F, capital G. So there's going to be two components to this. So we have the first component from F, small G, which is negative L over two, because I'm the radius, this moment arm here is L over two. And since this acts in the negative y direction, we're interested in this radius here along x. So this is going to be times sine of 45. Sine of theta sorry, leave everything in terms of the variables. And then we multiply this by the force, which is the mass of the rod, which is above the rod, which is 0.6 kilograms, always no matter what length times G is going to be negative, because it makes everything rotates clockwise. Okay. Next, we're gonna move on to F capital G, which is negative l sine theta, times that mass of the disk times G. Okay. And again, this is negative because it makes everything rotate clockwise. Okay. So now, this equation relates alpha with L, but we still have this unknown term which is ay ay ay. Everything else is known. Okay, so we need to solve for I about A. So since we have multiple components, we're going to look at each component separately. So first, we're just going to take I about a of this rod here. And then we're going to take it about a of this disc back here. So ay ay is going to be equal to so for the rod, remember, it's just one half m L squared, Ml squared, sorry, 1/3, sorry, 1/3 mL squared. This is simply for a rod about the end of a rod. Okay. Then we're going to add the disc. And so the disc, we have two components to I, which is first of all, the inertia about this point over here on the disk, the center of the disk, rotating in this direction, plus parallel axis to bring it back to a because it said, radius a, from that point, so we're going to have plus one quarter r squared, which is essentially for the disc, and then plus, and D squared, which is parallel axes, where this D is going to become L. So if I plug in L, and the values, I get the following that ay ay ay is going to be equal to 1/3 times times 0.6 grams L squared, plus one four times one kilogram, times 0.3 meters squared, plus one kilogram times L squared. And so this is now we have AA, just in terms of L, we could plug it back in, and we have an equation relating alpha and L. And with everything else being known, so we can directly solve for give given an alpha, which we are we can directly solve for L. So let me rewrote the equation in full and add in all of the numbers. So we have 1/3 times 0.6 kilograms squared, plus 1/4 times one kilogram times 0.3 meters squared, plus one kilogram L squared is equal to negative L over two sine of 45 degrees. The M of the rod is going to be 0.6 kilograms, times 9.81 meters per second squared. And I'm going to go on and you line here, minus l sine of 45 degrees times the mass of the disk, which are given is one kilogram times gravity 9.8 100 per second squared. And so now we can solve this equation because I'm sorry, I forgot to multiply by alpha over here. So that's i alpha equals to this sum of the moments, and everything is in terms of L. In since we're given an alpha of five radians per second squared, that's the maximum alpha, we're going to plug in alpha, and solve for L. And as you can see, this is a quadratic equation. So this is the resulting quadratic equation that we get. So we plug in alpha is equal to five radians per second squared, and we get the following negative six L squared minus 0.1125 plus 9.0177. l equals to zero. When we solve this, we get that l is equal to either 0.0126 meters or m l is equal to 1.49 meters. Okay. And between these two answers, the question asked to pick the most reasonable one, which is 1.49 meters, because this would be a very, very small ride. Okay, so this is our final answer. l is equal to 1.49 meters