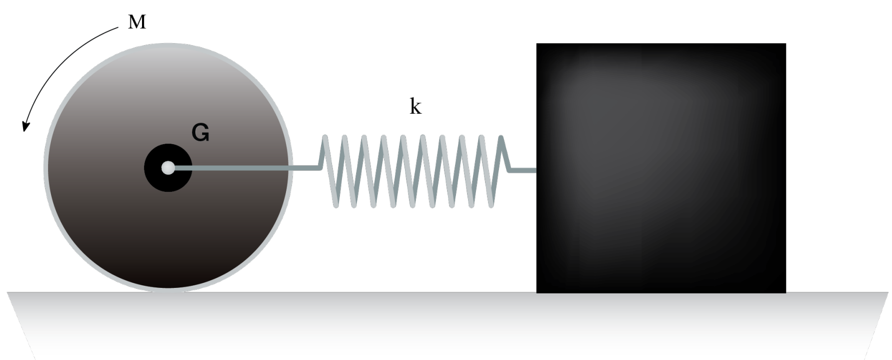
A lazy engineer is designing a robot to move things for him. He places a hub motor inside a 2.5 Kg disk such that a couple moment of $M = 2.943\ Nm$ is applied. If the attached package has a mass $m = 5\ Kg$ and the coefficients of kinetic friction and static friction for between all objects are $\mu_k = 0.2$ and $\mu_s = 0.4$ respectively, determine the angular velocity of the disk after its center of mass has travelled a distance $d = 0.5\ m$. Assume the disk rolls without slipping and the package does not tip. The disk has a radius $r = 0.3\ m$, the spring constant is $k = 100\ N/m$ and the spring is unstretched originally.



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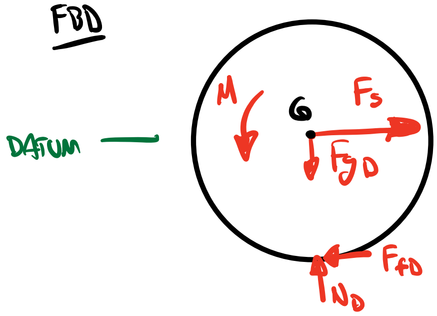

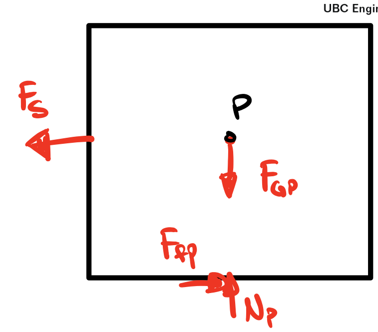

State #1 (resting)

$$T_1 = 0\ J \quad ; \quad V_1 = 0\ J$$

State #2 (final position G moved 0.5 m left)

$$T_2 = \frac{1}{2} I_{GD}\,\omega_D^2 + \frac{1}{2} m_G v_{GD}^2 + \frac{1}{2} m_P v_{GP}^2 \qquad V_2 = \frac{1}{2} k x^2$$

$$\sum F_{yp} = 0 \Rightarrow N_p - F_{gp} = 0 \Rightarrow N_p = m_p g = (5\,kg)(9.81\,m/s^2) = 49.05\,N$$

$$F_{fp} = \mu_K N_p = 0.2(49.05\,N) = 9.81\,N$$

$$\sum F_{xp} = 0 \Rightarrow F_{fp} = F_s \Rightarrow F_s = 9.81\,N$$

$$F_s = KX \Rightarrow X = \frac{F_s}{K} = \frac{9.81\,N}{100\,N/m} = 0.0981\,m$$

$$\theta = \frac{d_G}{r} = \frac{0.5\,m}{0.3\,m} = \frac{5}{2}\,rad$$

$$d_p = 0.5\,m - 0.0981\,m = 0.4019\,m$$

$$U_M = M\theta = (2.943\,Nm)\left(\frac{5}{2}\,rad\right) = 4.905\,J$$

$$U_F = \cancel{U_{FFD}}^{\,0} + U_{FfP} = F_{fp}\,d_p = (9.81\,N)(0.4019\,m) = 3.943\,J$$

$$T_1 + V_1 + \sum_{non\text{-}cons} U_{1\to 2} = T_2 + V_2$$

$$0 + 0 + 4.905\,J - 3.943\,J = \frac{1}{2} I_{GD}\,\omega_D^2 + \frac{1}{2} m_D V_{GD}^2 + \frac{1}{2} m_p V_{Gp}^2 + \frac{1}{2} K x^2$$

$$\vec{V} = \vec{\omega} \times \vec{r} \Rightarrow V_{GD} = \omega_D r = \omega_D (0.3\,m)$$

$$V_{GD} = V_{GP} \qquad I_{GD} = \frac{1}{2} m r^2 = \frac{1}{2}(2.5\,kg)(0.3\,m)^2$$

$$0.9624 = \frac{1}{2}\left(\frac{1}{2}(2.5\,kg)(0.3\,m)^2\right)\omega_D^2 + \frac{1}{2}(2.5\,kg)(0.3\,\omega_D)^2 + \frac{1}{2}(5\,kg)(0.3\,\omega_D)^2$$

$$+ \frac{1}{2}\left(100\,\frac{N}{m}\right)(0.0981\,m)^2$$

$$0.9624 = 0.05625\,\omega_D^2 + 0.1125\,\omega_D^2 + 0.225\,\omega_D^2 + 0.4812 \quad [J]$$

$$\hookrightarrow \boxed{\omega_D = 1.1\ rad/s}$$