Problem **20-R-WE-DK-15**

In this problem, we have a wheel, where we apply a moment, and this wheel starts spinning and it starts dragging a block along a distance and the block and the wheel are connected by spring, we need to find what is the angular velocity of the disk. After the center of mass G has moved point five meters to the left, given all of the properties of the system, so the block having friction with the bottom surface of spring connecting the two systems, and then a wheel rotating without slipping at the bottom. Okay. So what we're going to do is we're going to first draw a freebody diagram of the system, just to see all the forces. So our first free by we're gonna split it into two, obviously, our first freebody diagram being that of the wheel. And we're going to draw all of the forces. So the first force is the gravitational force, and that's going to occur at the center of gravity. And this is at g of D, obviously. And then we have a normal for us at the bottom with Mimi cold and of D. And then we have a friction force directly in this direction, called Ff of D. Okay, and then we have the force of the spring net pulls towards the right, and this will call the force of the spring F of s. And then here, a moment is applied in the counterclockwise direction, this moment, spin is what is essentially what we're the energy that we're putting into the system to make this whole system rotate. Okay. Now, if we move to the block, on the right, we have a rectangle. And we have our gravitational force, like always, this one's a different one than what the wheel has. So this is going to be f g of P. And then we have, again, a normal force that points upwards and of P. And then we have a force of friction, which points to the right because the block is sliding towards the left, which is f f of P. And then we have our spring force, which pulls, actually, we can draw it on the side, which pulls everything like this. And this is the same magnitude as f of s, because the spring has can just carry one force, and it's whatever force and on one side is carried by the other side. Okay, so now that we have a freebody diagram, it's going to simplify, we can see what forces are acting and you'll see why we really need that later. But this is still a work energy problem. And so with work energy, we need to define two states. And then compare the energy equate the energies between the two states in adding or subtracting any work that is non conservative that is lost out of the system or put into the system. So in this case, the initial states were given everything is resting on. So initially, there is no kinetic energy. And then after a while, after we put this, we add this moment, and the system starts moving after the energy has moved five meters 0.5 meters towards the left, then we're going to be left with some kinetic energy. So the energy that we've added into the system from the moment has been turned into kinetic energy, both of rolling for the for G, and then P is going to be sliding, but some of this energy will also be lost because there is friction. So as you can see, this problem is quite complex. And there's many aspects to it. And to keep track of everything, it's really important that we draw this freebody diagram and then we calculate separate the two states and calculate all of the energies for the two states. So we're going to start with state number one. And this is when it's resting. So initially, it's resting. So T one is going to be equal to zero joules, no energy, no kinetic energy. And we're also going to draw our datum right over here. So right around long the center of gravity. So in this case, we're gonna have no potential energy. So I'm gonna Write it next to it v one is equal to zero tools. Okay, because the spring is unstretched, so no, no energy stored in the spring. And there's no, everything is aligned with the datum, so there's no energy, there's no potential energy, gravitational potential energy, then we have state number two. Okay, and state number two is the final position, G, moving 0.5 meters left. Okay, so in this final position, as I said, we're gonna have some kinetic energy. So the potential energy is still going to be zero because nothing has moved up or down in terms of gravitational potential energy. So we can scrap the gravitational potential energy portion. But we are going to have some spring potential energy. So we'll leave that for later. For now let's just look at the kinetic energy. So T two is the kinetic energy at two M is going to be equal, we're going to have some kinetic energy due to the wheel rolling. So one half i g d omega d squared. So this is the energy the kinetic energy due to the rotation of G, then we have some kinetic energy due to the translation of G, so one half m g, v, G, D squared, and then we have some kinetic energy due to the translation of the block to the right, so block p, so one half m p, V, g p squared, and again, GG means the center of gravity of D, GP, the center of gravity of P. Okay, same thing with GD, here, it's I about g of D. Okay, always the center of gravity. So this is going to be your kinetic energy for state number two. Okay, now, we don't know these expressions, we don't know omega d, no, no big deal. We don't know BGP, so we're gonna have to work them out. But essentially, what we're looking at this problem, when everything starts moving, you're going to start with everything stationary. So initially, we have a static friction coefficient, which, as you can see, is larger than the kinetic friction coefficient. So as this applies a moment, this is going to start rolling, this spring is going to start stretching, but this block here will not be moving. Okay. And that is because this block here has a friction force that is holding it in place. Now, as this spring is stretched more and more, this force here increases more and more. So the force pulling, and at certain point, this force here will overcome the friction force that points backwards. And once that is overcome, this block starts moving. Okay. Now, once it starts moving, we don't have a static friction coefficient anymore, but we have a dynamic friction or kinetic friction coefficient, which is lower. So that means this friction force for the same normal force will be lower, meaning that this block is actually going to accelerate a little bit and this spring is going to compress a bit more. So the spring is going to start compressed like this, then it's going to get longer. And then after what after this starts moving, it's going to shrink and get a bit shorter, it's still longer than the original length, but a bit shorter, because of that change in friction coefficient. Okay, um, but here, we're not looking at the transient response. So we're not looking at every point in time, we're just looking at the difference between the beginning and the end. So this will not really matter too much. Because one we're looking at the our second point, that is solely going to involve the kinetic friction coefficient because everything will have already been moving. And so this transition doesn't really matter on but you do need to be aware that this does occur. Okay. So at state to the package will move at a constant velocity. Okay, so when the cons when we have a constant velocity that means that the acceleration is going to be zero, there's no acceleration. So when we do a force balance for these freebody diagrams, we know that there will not be an acceleration. And so we can do our moment balance, some force and force balances and add everything, equate them to zero, which is what we're going to do next on to solve for some of these unknowns that we don't know yet and that we'll use later. Okay, so let's start with a sum of forces in y for P. Okay, so some, some forces in y four p is equal to zero as I mentioned, this yields NP minus f g p is equal to zero. Okay? So FTP is just m, p g, so we can equate this to NP being equal to m, p times G. Okay, and this is pretty self explanatory. So if we plug everything in, we get n p is equal to five kilograms times 9.81 meters per second squared, which is equal to 49.05 Newtons. Okay, so that's going to be NP. And again, we're going to need these values for our friction forces, which we'll then use later, okay. Then we have f f p being equal to mu k and P. So this is just the friction equation. So FFP is equal to mu, okay, because everything, as I mentioned, is moving times and of p, which is equal to 0.2 times 49.05 Newtons. So our F p is going to be 9.81 Newtons. Okay. So 9.81. Okay, so then we have our sum of forces in the x direction P, and this is going to also be equal to zero. Okay, so this yields f, p is equal to f of x or the spring for us. So this means that f of s is equal to 9.81. Okay, now, the force of the spring is also equal to f of s is equal to k x. Okay, so this x here represents the change in length of the spring due to the force. So, if you apply force to spring, it extends, okay, so that x represents that extension of the spring. And since we were told that the spring starts from a from its original unstretched length, then that, that whole delta x is the whole change in length. And we don't have to subtract l not. Okay. So, we know F of s, because we've just calculated it. This is 9.81 Newton's, we know K. So we can actually find x. So x is equal to f s over k, which is equal to 9.81 mutans over k, which is zero point, sorry. 100 Newtons per meter. So x is actually equal to

0.0981 meters, hey, so this is how much the spring stretches. Okay, so if the spring stretches by that much, that means that the package moves 0.981 meters less than the disk. Okay? That's because if you go into this diagram over here, if this thing moves, and for the first 0.098 meters, this doesn't move, then this will have moved more than this. Okay? So when we count point five meters of movement for this, that doesn't equate 2.5 meters of movement for this because It goes into stretching that spring, which takes up a bit of distance, distance. Okay. So that's why we need to keep into account that delta x, which is, is very sneaky. Next, we're going to look at the rotation of that of G. And how much distance is trapped how much rotation occurs after g travels point five meters. So if we look at this circle here, again, it's pinned over here at the bottom, because there's, we're told there's no slip here, on so if this has to travel on point five meters, we know that this radius crop omega cross r is the distance is omega theta cross r is the distance traveled by this point here. So since everything is orthogonal, we can just say that theta is going to be equal to d over R. Okay, where d is the distance that is traveled, and r is the radius of the front of the point of no rotation, which is the bottom to G. Okay? So it's again the radius of that circle of key. So, theta is going to be equal to 0.5 meters over 0.3 meters, which is equal to five over two radians okay. So, this is how much v rotates and this will be useful later on when we calculate the work due to the moment okay. So, if we won't go back to this, we said that this was how much was traveled, how much less the block traveled with respect to g. So, we can actually solve for the distance of that P travels okay. So, the distance that P travels is actually equal to that 0.5 meters minus 0.0981 meters, which equals to 0.4019 meters okay. This is a simple subtraction. So, again d is the distance that g travels. So, this is again we can see dg dp here is the distance that P travels which is d g minus this spring stretching time, okay. So, now, we have found the forces, so, the friction force. So, from this diagram here we have found this friction force which we need, because this creates a work we have found the distance this friction force travels, we have found the, so, we found v g which is the linear velocity of this broad block, which gives us some kinetic energy a state to VG here sorry, we p here is different, is the same sorry. And then we have found we need to find omega, which is energy related creates energy related to this rotation over here. And this is all included in these terms, then we found here is friction for us here. There's no friction. So this FFD does not create any, any moment because there's no slipping, right? So if there's no slipping that force doesn't go along a distance. So this f of f d doesn't add work or dissipate work. And so now we're ready to add everything up and do some calculations. So first, let's calculate the work due to the moment. So U of M. So this is energy that we're putting into the system is equal to the moment or the couple times theta, which is equal to 2.943. Which is given in the question and it's a Newton meters. times five over three or two sorry, radians. And this is the angle theta that we just learned. So this is going to be equal to 4.905 joules, then we can find the work that is lost or dissipated due to friction, this will call you. And this, as I mentioned, is a combination of all the friction forces, you have of D plus you have P. But as I mentioned, this doesn't travel a distance, so it's zero, it is just going to be USF with P. And again, it's a force times a distance. So it's going to be force F F of P times dp. Okay, and as the I mentioned, we just found dp, it's not the same distance traveled as G, because the spring stretches. And so this is going to be equal to 9.81 Newton's times 0.4019 meters, which is equal to 3.943. jewels. Okay. And these are all the energies that are added are dissipated. So non conservative forces. Now we're ready to take our full sum of energies and equate state one to state two, while adding those non conservative forces. So T one plus v one is plus the sum of nonconservative from one to two is going to be equal to on T two plus v two. Okay. And so we can say that T one and v one are both zero, because there's no there's none of them. So zero, plus zero, then we calculated these two nonconservative works, but we have to watch for the signs of these. So first, let's look at u, u m is work that we are adding into the system. So it's going to be positive. So plus 4.905. Okay, joules. So this work here we're adding into the system. So that's why it's positive, though force due to friction is dissipative work. So that's why this one has to have a negative sign negative 3.943 jewels.

Okay. So that's a negative work. Because it's dissipated, it's out of the system. And this is going to be equal to we said there's nothing no other works, it's going to be equal to one half i g d squared omega d squared plus one half m d v g d squared, plus one half p v g p squared plus one half k x squared. So like I mentioned, sorry, I should have added it here. B two is going to be equal to one half k x squared, because potential energies still the same, but we do have that extra stretch in the spring. Okay, so this is just this whole term is I two plus T two plus v two. Alright, so now, we can start plugging things in and solving for omega, but as you can see, in this equation, we actually don't have everything in terms of omega. So the masses are given but VG D BGP, we don't know these quantities. So, we can actually solve for VG D in terms of omega, because we know that this is pinned over here. So if this is pinned, we can find velocity of G which is this center point here. And p here, we can find it by using omega equals to r v equals r cos omega. Okay, so if we go down here, we can use v is equal to omega cross r But since we know everything is orthogonal, we can just directly multiply. So V, G, D is going to be equal to omega d times R, and this r is the distance from here to here. So it's again our the radius of that circle. So we can actually plug this directly in with our being equal to 0.3 meters. So let's maybe go omega d times 0.3 meters. Now we can also say that the key D is equal to b GP, because we said nothing is accelerating, everything must have the same velocity. And velocity of this must be equal to this. Okay, so then we can substitute this equation here, omega d, into here and into there. So we eliminate those linear velocity terms, and everything is in terms of omega. The last thing we need to do is we need to find AI, which is one half and I r squared. So that's just inertia, I G, D is equal to one half r squared, which is equal to one half times what? 2.5 kilograms times 0.3 meters squared. Okay. So after this, we have so we have everything in the equation, and everything is in terms of omega d. And we can solve this quadratic equation and solve for omega d. So let me plug everything in and solve for the final answer. 0.9624 is equal to one times one, plus 2.5 kilograms, is 0.3 meters squared. Omega d squared plus one half is 2.5 kilograms, times 0.3 omega d squared plus one half times five kilograms, this is the block to the right, times 0.3 omega d squared plus one times 100 Newtons per meter, times 0.0981 meters squared. Sorry, the square goes outside of the bracket. Like that. So if we solve this equation, we get 0.9624 is equal to 0.0 point 05625 omega d squared plus 0.1125 omega d squared plus 0.225 mega d squared plus 0.4 H, one, two. And all of these terms are in joules, because this is energy. And if we solve this quadratic equation, we get that omega d is equal to 1.1 radians per second. And this is our final answer.