

## Problem 8

In this problem, we have a reel of mass 15 kilograms, it is resting on two rollers. And it's initially at rest, we apply a force of 400 with the rope attached at  $r_1$ . And given the radius of gyration, we are asked how many revolutions must the wheel complete to achieve a final angular velocity of 30 radians per second. So this is a work energy problem. And we're starting from the state where the reel is at rest. And we need to add a certain amount of work. So force  $P$  times this times the distance to achieve a final angular velocity of 30 radians per second, we're also assuming the rollers are frictionless. And they we can neglect their mass in terms of increasing their kinetic energy as well. So we're going to first look at the geometry. So we're going to be pulling this rope. So this force  $P$  is constant. And applying this force along a distance, again, is going to give us the work that we input into the system. Now the distance that this rope travels, this, we can find this based on the geometry, right. And this is going to be related to how many revolutions this wheel makes, right, because if this wheel makes one revolution, this distance here that the rope travels, is going to be one circumference, right? And circumference with  $r_1$ , because this is where the rope is attached. So we can come up with a function for the displacement of that rope,  $D$ , this is going to be equal to  $2\pi r_1$ . So this is the circumference. And then we add in the end term. And this  $n$  is essentially the number of revolutions, right, because we are asked in the problem to find how many revolutions must the wheel make to complete this angular velocity. So this is just how many times this circumference does this rope have to travel and again, one circumference is one revolution. So this is the number of circumstances or the number of revolutions. So this is what we are trying to solve for. Right. And again, this is going to have a unit of meters because it's a distance, and we can plug in  $r_1$ . And we get that  $d$  is equal to 1.257 N meters. Okay, so this is our distance, now, we have to find the work that's expended, right, the work that we add into the system. So our energy equation, we're going to start with the initial kinetic energy  $T_1$  plus the work that gets us from 2 to 2, this is going to be equal to  $T_2$ , right? I'm ignoring the

potential energy because there's no change in potential energy in the system. So we just ignore it, we just keep track of the kinetic energy and the work. So we know that  $t_1$  is zero, right? Because this is given into question, because we do not have any velocity any rotation, initially, because it starts from rest, right? So we just have these two terms over here, kinetic energy final and the work required to get us from the initial state, the stationary state to the final state where we're rotating. Now, the work from one to two, we discussed is just the force times the distance. So work from 1 to 2 is equal to the force, which in this case, is the force  $P$  times that distance  $d$ . Right? So this is going to be equal to 400 Newton's times 1.275 N meters. And this is again going to have units of joules, right. So this is the work we have an expression for the work now we have to find an expression for the final kinetic energy, right? And as you remember, the kinetic energy has the form the following formulation, one half  $I \omega$  squared. Now I have this is just in terms of  $\omega$  because this system is not translating the CG is stationary, right? It's just staying still over here. It's just going to rotate. That's why I just have the one half  $I \omega$  squared term. And we need to figure out what  $I$  is. So the inertia is equal to so this is we're going to we're given the radius of gyration. So  $I$  is equal to  $m k$  squared. And if we plug in 15 kilograms, times 0.6, which is  $k$ , the radius of gyration, which is unit meters, and then we square that, it gets us an  $I$  of 5.4 kilograms, meters squared. Okay. And, again,  $T_2$  is going to be equal to one half,  $I \omega$  two squared. And this is going to be equal to one half times 5.4 kilograms, meters squared times 30 radians per second squared, right? Because this is the final angular velocity, which were given in the question over here. So we can now solve so this is just a number, right? We can take this number, so we have this expression here for  $T_2$ . And we have this expression over here for the work from 1 to 2. And this is just an equation in one unknown, which is  $N$ , right? The number of revolutions or the number of circumferences, so we can directly solve the following equation. So putting everything together, we get 400 Newton's times 1.257 N meters, is going to be equal to one half 5.4 kilograms, meters squared times

30 radians per second squared, all squared, and we can solve for N. So N is going to be equal to 4.83 revolutions and this is our final answer.

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