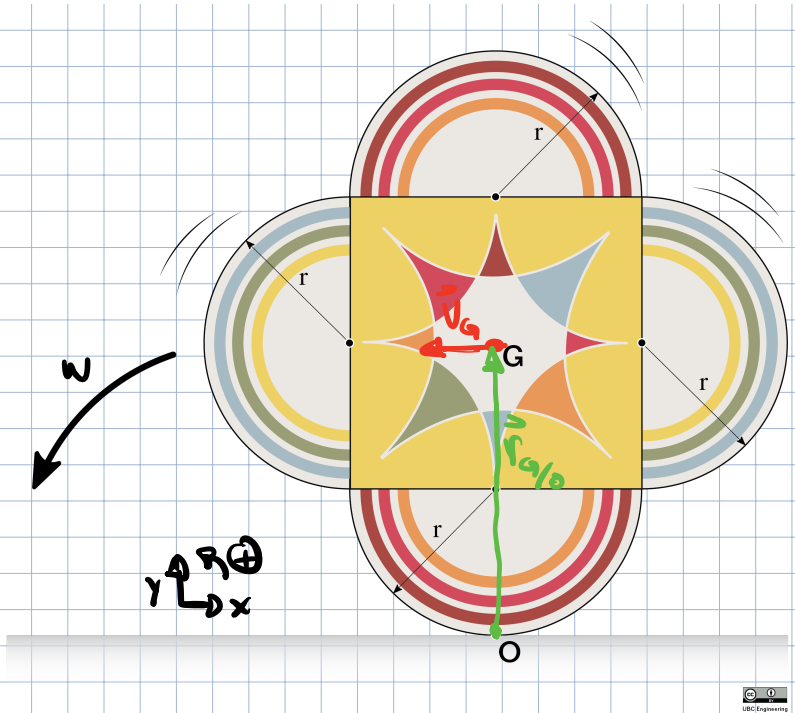
A toy is made of a square with 4 semi-circles of radius 14 cm attached. It has a mass of m = 400 g and a radius of gyration k = 20 cm about its center of mass G. If it rolls to the left without slipping and has 3 J of kinetic energy at this moment, find the linear momentum of the toy, the angular momentum about its center of mass G and its angular momentum about the point of contact with the ground O. At this instant, the center of mass is directly above the point of contact with the ground.



$I_G = mk^2$

$= (0.4\,kg)(0.2\,m)^2$

$= 0.016\ kg\,m^2$

$I_O = I_G + m\ell^2$ (where $\ell = 2r$)

$= (0.016\ kg\,m^2) + (0.4\,kg)(2 \cdot 0.14\,m)^2$

$I_O = 0.0473\ kg\,m^2$

$T = \frac{1}{2} I_G w^2 + \frac{1}{2} m v_G^2$

$T = \frac{1}{2}(0.016\ kg\,m^2)w^2 + \frac{1}{2}(0.4\,kg)4w^2(0.14\,m)^2 = 3\,J$

$\therefore w = \sqrt{\dfrac{3}{((0.5)(0.016\ kg\,m^2) + 2(0.4\,kg)(0.14\,m)^2)}}$

$w = 11.26\ rad/s$

$\therefore \vec{w} = 11.26\ rad/s\ \hat{k}$

$|\vec{r}_{G/O}| = 2r$

$\vec{v}_G = \vec{v}_O + \vec{w} \times \vec{r}_{G/O}$

$\therefore v_G = w|\vec{r}_{G/O}|$

$v_G = w2r$

$\therefore \vec{v}_G = \vec{w} \times \vec{r}_{G/O}$

$\vec{v}_G = \vec{v}_O + \vec{w} \times \vec{r}_{G/O} = 11.26\,\hat{k}\,(rad/s) \times 2(0.14\,m)\hat{j} = -3.15\,\frac{m}{s}\hat{i}$

Linear Momentum: $\vec{L} = m\vec{V}_G = (0.4\,kg)(-3.15\,m/s\,\hat{i}) = -1.26\,\frac{kg\,m}{s}\,\hat{i}$

$$\boxed{\vec{L} = -1.26\,\frac{kg\,m}{s}\,\hat{i}}$$

Angular Momentum: (about G) $\vec{H}_G = I_G\vec{\omega}$

$$= (0.016\,kg\,m^2)\left(11.26\,\frac{rad}{s}\,\hat{k}\right)$$

$$\boxed{\vec{H}_G = 0.18\,\frac{kg\,m^2}{s}\,\hat{k}}$$

Angular Momentum: (about O) $\vec{H}_O = I_O\vec{\omega}$

$$= (0.04736\,kg\,m^2)\left(11.26\,\frac{rad}{s}\,\hat{k}\right)$$

$$\boxed{\vec{H}_O = 0.53\,\frac{kg\,m^2}{s}\,\hat{k}}$$