Problem **12**

In this problem, there's a toy that's made with four semi circles of radius 14 centimeters. And they're attached to a central square. This toy has a mass of 400 grams and radius of gyration of 20 centimeters about its center of mass G. This is rolling to the left, about o, and it's not slipping with respect to the surface and has a kinetic energy at this instant of three joules. We're asked to find the linear momentum of the toy, the angular momentum about its center of mass G and the angular momentum about the point of contact with the ground or the instantaneous center of zero velocity o. And at this instant, the center of mass is directly above the point of contact with the ground o. So what we need to do is we first we need to analyze the problem, the geometry, so we have these four semi circles of radius R attached to a square. So the square is going to have a height and a width of two R. And we're going to start with what we're given in the question. So in the question, we're given that it has a kinetic energy of three joules. So from this, we can determine the angular velocity of which it's spinning with, right? Because that is what we'll need to calculate the linear and angular momentums. So with, we're going to first come up with a formulation of the kinetic energy and try find omega. So the angular velocity didn't determine either the velocity and the angular velocity that we've already found, to find the different linear and angular momentum. So let's come up with an expression for the kinetic energy. So we know that t is going to have two components, right? One due to the translation of the center of gravity, and one due to the rotation about of this object, right? So the first term is going to be dealing with the rotation. So this is one half IG omega squared plus one half m V g squared. So again, this this deals with the rotation because it has omega, and this deals with translation, because it has V, right? And again, this is about the center of gravity, G, so this point over here, so we have to find IG. So inertia about G, and then the velocity of the center of gravity, G, which again, depends on omega, right? So let's first find IG. So IG is going to be equal to m k squared, right? Because we're given the radius of gyration, which is 20 centimeters. And we know the maths so we can find IG. So IG is going to be equal to 0.4 kilograms, times 0.2 meters, because it was 20 centimeters squared. And the IG is meant to be equal to 0.016 kilograms meters squared. So now we know IG we're trying to solve for omega, right? We can solve for omega, or VG doesn't matter either or because we have an equation that links omega and VG, right. So we have the following equation, V G, is equal to V naught plus omega cross r of G with respect to O. And these are all vectors. Right? So we we know that this point G here will have a velocity to the left, and this is V G. And we know that there is, since we're starting from this point here, there's a radius or a distance called R of G with respect to O. That goes like that. And lastly, we can get rid of this first term here V naught, that zero because we know we're told in the question that this point is not slipping, and the surface is stationary. So therefore, that point on the object is also stationary, not translating with any velocity. So we know that V naught is zero. So we can condense this equation down to simply V G equals to omega cross r of G with respect to the other thing and this is omega right? The other thing is we know that all these three guys factors are perpendicular to each other. So one points in the y direction one points in the x direction and then omega, in this case is rotating counterclockwise, which means is positive, so it's pointing out of the page, right? So these are perpendicular. When we have perpendicular vectors, we don't, we can turn this into a scalar equation, because it directly turns into the multiplication of the magnitudes, right? So from this, we can determine that V G is equal to omega r. And again, these are the magnitudes and this r is this is the magnitude of this green vector that I just drew here. So that's, so we can write it onto the side here, r is going to be equal to two r. So r, this is going to be the magnitude of r g with respect to right, and let me add that into here as well. That's going to be two r because it's one radius plus another radius here. Okay, so we know that VG is equal to omega two r. And we can plug this into this equation here. And as I said, it doesn't matter what we solve for, we can solve for omega or we can solve for VG, because with this equation, we can relate omega and VG. In this case, we're going to solve for omega. So we can rewrite the following equation to one half t equals to one half 0.016 kilograms meters squared times omega squared plus one half, the mass here is going to be 0.4 kilograms, times V G, which is four omega squared times r squared, and r squared, we are given that r is 0.14 centimeters meters, sorry, this is squared. So with this, we have an expression for the kinetic energy T. And that relates T and omega, right. So this here is going to be equal to three joules. So we can solve for omega by solving the quadratic equation. And we get the following. Omega is equal to square root of three divided by 0.5 times 0.016 kilograms, meters squared, plus two times 0.4 kilograms, times 0.14 meters, and an all squared. And you take the square root of this. So when we solve for this, we got that omega is equal to 11.26 radians per second. And if we look at this, we can see that omega the vectorial form of omega, so the vector is going to be 11.26 radians per second. And since we said this is rotating, counterclockwise, we're assuming of the following coordinate system x is positive to the right, y is positive upwards and a counterclockwise rotation is positive using the right hand rule, therefore, this is going to be in the positive k hat direction. Now, once we have omega we can start looking at the linear momentum first. So linear momentum so we know that L, going to be equal to m V G. Right, where m is the mass and V G is the velocity of the center of gravity. So we know when we rewrite this neatly, we know that the maths is 0.4 kilograms. And we have found an expression for VG right. We know that V G is two omega r, where if we rewrite this vectorial equation over here, V G is equal to omega cross r of G with respect to O, right? Because we know that this is a cross product. So we can compute this cross product, or we can simply look at this image over here. And we know that omega or VG, sorry, will point in the negative x direction, right based on the cross product. Or you can compute the cross product and see that you get a vector pointing in the negative x direction. So what we do is we plug in this value, which VG is going to be equal to negative 3.15 meters per second in the i hat direction, right. So I just what I did here is I plugged, I multiplied omega by r, because they know these two vectors are already perpendicular due to the cross product. So sorry, omega, which points out of the page, and R, which points in the positive y direction. So I multiply the two magnitudes. So omega is what we just calculated 11.26. And R is just two r right, because it's an magnitude to our G with respect to O, which is two R, and that gives me 3.15. And then it's in the negative I hat direction due to the cross product. So when we multiply everything together, we get that this equals to negative 1.26 kilograms, meters per second in the i hat direction. So L is equal to negative 1.26 kilograms meters per second in the i hat direction. And again, remember of the vector, so you need to add a direction. Next step, let's move into the angular momentum. So we're going to start with G. Because we have just calculated everything about G, we know Ig. So we're going to start with finding angular momentum about the center of gravity, G. So what we need to do is write down the expression. So H G, so this is the angular momentum about g is going to be equal to IG times omega. And again, we have just found the vector for omega, so we can directly use this, whereas here V G, we actually have to solve. So let me actually write out the expression for VG just for clarity. So V G, is equal to V naught plus omega cross r of G with respect to 0.0, which is going to be equal to 11.26 k hat radians per second crossed two times 0.14 meters in the j hat direction, which is equal to negative 3.15 meters per second in the i hat direction. So this is what you see this coming. Right. But now here, we can just use omega which we have just determined from this equation over here. So we know that IG we've just calculated up here is 0.016 kilograms meters squared. And we know that omega is equal to 11.26 radians per second in the k hat direction. So we know that h g will be equal to 0.18 kilograms meters squared per second in the k hat direction. And again, really important that you remember to add the unit vector for the direction okay. Now we're gonna have to do In the last part of the problem, which is finding the angular momentum about o. So Angular angular momentum about o, so we know that H O will be equal to Io omega, right. And Io is not the same as IG, right, so we need to find Io. So, the way we find Io is by using IG, so we can say that Io is equal to i g plus m r squared, so we're using parallel axis. And let me change r to m L squared, where l is the distance you're moving away from the center of gravity, which in this point is going to be two r right. So this here is going to be two r, L is to r. So, we can actually plug in these values. So we have 0.016 kilograms, meters squared plus m, which is 0.4 kilograms times two times 0.14 meters squared, and I naught is going to be equal to 0.04736 kilograms meters squared. And we have an expression for I not. So, this is equal to 0.04736 kilograms meters squared times omega, which is 11.26 radians per second, the k hat direction, so, H naught will be equal to 0.53 kilograms meters squared per second in the k hat direction. And this is our final answer