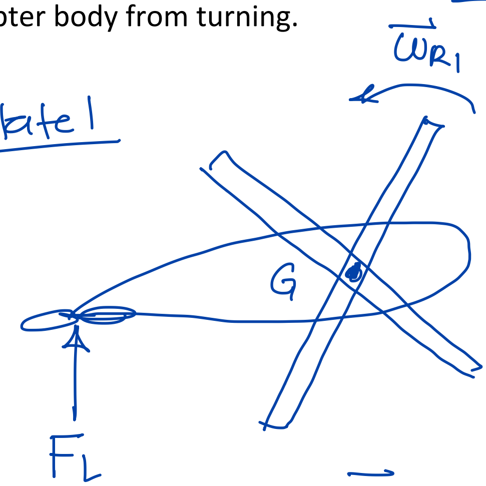
A helicopter tail rotor is used to prevent unwanted rotation of the body of the helicopter when the main rotor changes speed. Assume that the four main rotor blades are each long thin rods with length of 5 m and mass of 30 kg. Assume that the helicopter body has a mass of 750 kg and a mass moment of inertia of 1300 kg-m2 at its centre of gravity, located vertically in line with the main rotor. The distance between the main and tail rotors is 6 m.

state1 state2

a) If the tail rotor is functioning, and the angular velocity of the helicopter body remains at zero before and after the main rotor changes speed from 200 rpm to 300 rpm, find the final horizonal velocity of the helicopter body (starts from rest).
b) Assuming the main rotor change in speed from 200 rpm to 300 rpm occurs in uniformly over 8 s, find the force exerted by the tail rotor to keep the helicopter body from turning.

State1



$\vec{\omega}_{B1} = 0$

$\vec{V}_{G1} = 0$

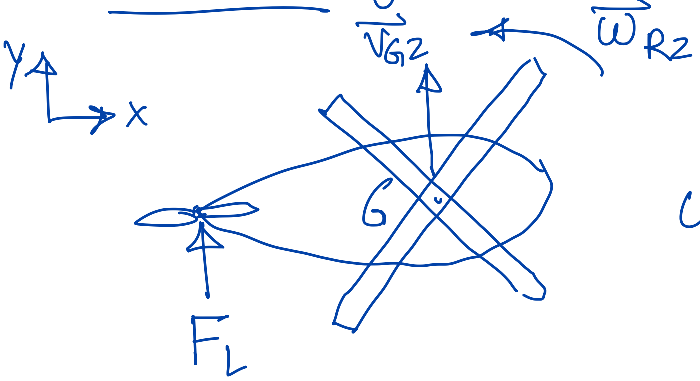
$$\vec{J}_{y1} = 0 \quad \vec{J}_{x1} = 0$$

$$\vec{K}_{G1} = I_R \vec{\omega}_{R1} \left(+ I_B \cancel{\vec{\omega}_{B1}}^{\,0}\right)$$

$$\vec{\omega}_{R1} = 200 \frac{rev}{min} \cdot \frac{1\,min}{60\,sec} \cdot \frac{2\pi}{1\,rev}$$
$$= 20.9\ rad/s$$

$$I_R = 4\left(\frac{1}{3} m_R l^2\right)$$
$$= 4\left(\frac{1}{3}(30kg)(5m)^2\right)$$
$$= 1000\ kg\text{-}m^2$$

State 2 just after rotor changed speed.



$\vec{\omega}_{B2} = 0$

$\vec{V}_{G2} \neq 0$

$\vec{J}_{y2} = m_{TOT} \vec{V}_{G2}$

$\vec{J}_{x2} = 0$

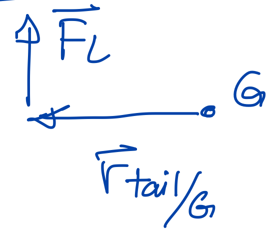
$$\omega_{R2} = 300 rpm$$
$$= 31.4 rad/s$$

$$\vec{K}_{G2} = I_R \vec{\omega}_{R2}$$

linear impulse-momentum (in y-dir):

$$\cancel{\vec{J}_{y1}}^{\,0} + \int_0^t \vec{F}_L \, dt = \vec{J}_{y2} = m_{TOT}\vec{v}_{G2}$$

angular impulse-momentum (about G):



$= 6\,m(-\hat{i})$

$$\vec{K}_{G1} + \int_0^t \vec{r}_{tail/G} \times \vec{F}_L \, dt = \vec{K}_{G2}$$

$$\vec{K}_{G1} - r_{tail/G} \underbrace{\int_0^t F_L \, dt}_{m_{TOT}\, v_{G2}} \hat{k} = \vec{K}_{G2}$$

$$I_R \omega_{R1} \hat{k} - 6\, m_{TOT}\, v_{G2} \hat{k} = I_R \omega_{R2} \hat{k}$$

$m_{TOT} = m_B + 4m_R$

(a) $$v_{G2} = \frac{-I_R(\omega_{R2} - \omega_{R1})}{6\, m_{TOT}} = \frac{-1000\,kg\cdot m^2\,(10.5\,rad/s)}{6\,(750\,kg + 4 \times 30\,kg)}$$

$$\boxed{\vec{v}_{G2} = -2.0\,m/s\,\hat{j}}$$

(b) from linear momentum, $t = 8\,s$

$$\int_0^8 F_L \, dt = m_{TOT}\, v_{G2} = (870\,kg)(-2.0\,m/s)$$

$$F_L(8-0) = -1745\,N\cdot s$$

$$F_L = \frac{-1745}{8} \Rightarrow \boxed{\vec{F}_L = -218.1\,N\,\hat{j}}$$