Problem **20-R-IM-DK-9**

This problem, we're looking at a rod that is suspended. The rod has a length l. And we're, we apply an impulse at the bottom end of the rod. And we're asked to find what is the vertical location of point O, where the rod only rotates? So essentially what is where is the iczv, instantaneous center of zero velocity, where the rod only rotates and does not translate. Okay? And so here that's denoted as Oh, and the distance is going to be from the bottom to the top. And that's going to be the distance that we're looking for. So this distance l. Here, actually, oh, is the whole distance, we're just looking for the distance from the bottom to point out, which is the ICZV all right. So this is an impulse and momentum question. And again, this question doesn't have any numbers, it's just theoretical. So we're just looking, we're not solving for a numerical solution, we're going to get the solution in terms of L. Okay, so we're gonna start with momentum, and bonds impulse and momentum balance about x. And then, and then we'll go on from there. Okay, so I'm going to draw in the coordinate system for this rod over here. So I'm going to draw the freebody diagram on the side. And I'm going to draw the forces in. So this is the rod drive better. And this rod is going to have first of all a gravitational force at the center of gravity, fg. Then we have the tension force Ft, which pulls up on it. And then we have this impulse. And we're gonna call this I. Okay. So again, this is B at the bottom, then we have O, children and red somewhere over here. And then we have a at the top. And then this here is the center of gravity, G. Okay. So for this, since we're applying that impulse at the bottom, we are and this is only in the linear direction, our sum of momentum will look like this. So in the so again, x is going to be this way, y positive rotation counterclockwise. So the in the x direction, we have the mass of this pendulum, times v Gx at the beginning, plus the sum of the impulses. So from time one to time two, f of x in dt is going to be equal to mass times VG, x two. So again, these two are the velocities along x of the center of gravity. This is the impulse. Okay? So the impulse we can start the beginning there is zero velocity, so we're going to approximate this to be zero. Plus, this is just the impulse, right? So we're just going to call that I. And this is going to be equal to m vGx two. Okay, so we have I being equal to m Vgx to next we're going to do it. We're going to use angular velocities. And we're going to do it with respect to the moment. So here we have instead of again, velocities, we have angular velocity, and instead of mass, we have the mass moment of inertia. So IG omega one plus the sum over the integral from t one to T to a moment, g in dt is equal to, again, I G omega two. So just like before, we know that this term goes to zero because omega one is zero, so we have zero plus Here we have again, the only thing is going to be that impulse. So that force that we apply at the bottom here. So this impulse is just a force over us time. So force times the time. And so to for that, for us to create a moment about the center of gravity, which is located at the center here, we just need to multiply that impulse by this distance here, right? Because this is perpendicular to this radius. So the cross product is the whole product. And so we just take the impulse times the, that length, which is L over two. Sorry, let me fix this L over two, and that's going to be equal to i g omega two. Okay, so we can actually plug in i g to what it is. So it is going to be one over 12 M L squared, and then we have omega two. Okay. So here, we can actually solve for AI. So we isolate for I, and then we plug in the impulse into here to get an expression for VG x two. Okay, so um, this we're going to call one, this, we're going to call two. So from two, we get that i is equal to one over six and omega two. Okay. And then we take this and we plug it into one. So we take this and plug it into number one, and we get that VG x two is going to be equal to one over six. How omega two? We write that six better. Okay. Now, why did we solve for the velocity because today, we're trying to find the instantaneous center of zero velocity. Hey, so if we solve for a velocity, for example, the velocity of the center of gravity G. With that velocity, we know that we can find set this distance to be the distance that we're trying to solve for, then, we can try and find this instantaneous center of zero velocity. Okay, so let me draw in the velocities in green.

So we're going to assume that this velocity is a set amount, then VB is going to have a larger velocity. And we know that this should all cross through the instantaneous center of zero velocity. Because there we have a velocity of zero. And then higher than this, we have VA being in the opposite direction. So this is VA, this is VG and this is VB. Okay. So again, remember this line identifies the velocity at each point, setting this the velocity zero increasing with a higher radius and in the opposite direction increasing with a higher radius or higher distance from that point. And we're trying to find this whole distance here. So from B to O. Okay. Um, so let's go back here. We've solved for VG x two, and we are now trying to find VB. So we know that VB is just going to be equal to omega times R of B with respect to Oh, okay. And that is because again, this velocity is perpendicular to this radius here. And so that cross product gives us the full product. So now that we know this, we have we know two velocities, and then again, this is going to be omega two over here, because we're just looking at the at the final time point. So this here, is VB in terms of omega two, this here is VG x two in terms of omega two. And we can actually take the ratios to solve take a ratio and solve for that distance between oh and the bottom or between o and G and then we can find the respective other distance okay. So we know that VB over R of B with respect to O is going to be equal to V g x over r of B with respect to o minus L over two. Now, what is this saying? This is saying that omega two is equal to omega to the Omega two is equal to VB over r v with respect to O, which is this portion here. And then what is this saying that Megha two is also equal to V gx divided by r v with respect to o minus L over two. Okay, so our b with respect to O, I'm going to draw it in orange here is going to be this distance over here. R of B with respect to O. And if we subtract L over two, which is this distance, we get this little distance here. Okay, so here, that's why we have that equation over there. Okay. And again, this is this would be VG x two, and then this is BB two. And this is also BB two. Okay. So now that we have this equation, we can actually plug things in. So in here, we can plug this in terms of omega two. In here, we can plug this in in terms of omega two, and you can see that the Omega two is cancelled. And what we can solve for is our V with respect to o directly. And that will just be in terms of L. So we can we get the following omega b omega two times r b with respect to o over R of B with respect to O is going to be equal to one over six l omega two over R of B with respect to o minus L over two. So you can see that let me do this in red, this cancels and these two cancel, and we can directly solve for our fee with respecto which is going to be equal to one over six l minus r plus L over two. Therefore, r of b with respect to O is equal to three over two out and that is our final answer.