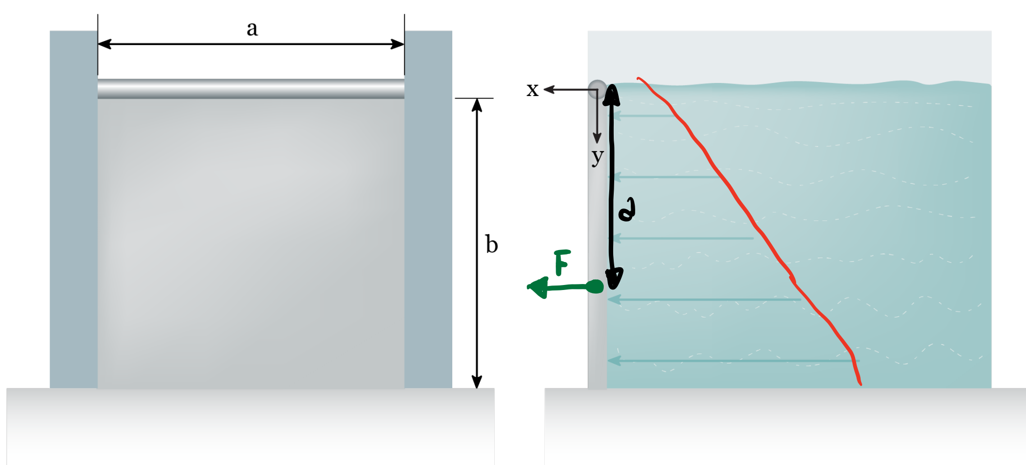
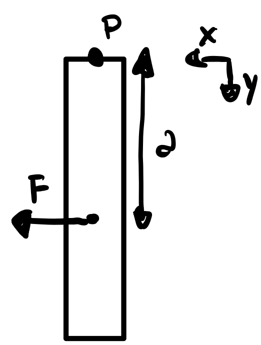
A salmon hatchery has a gate to release water whenever water levels get too high. The gate is normally locked into place, but when the water reaches. The top of the gate, the lock is instantly removed. While the water would flow out of the gate by itself, there is a pump located upstream from the gate to push water forward. The pump is old, so its power slowly ramps up with time. The gate consists of a slender rod in which a 20 kg thin plate is attached. The plate has dimensions $a = 1.5\ m,\ b = 2\ m$ and rotates about the slender rod as if it were a pin. Although there is a seal such that water may not get out, assume the contact between the plate and other surfaces is frictionless. If the gate is subject to water from the pump that applies a force distribution with a magnitude that is related to both the y-coordinate and time $dF = \left(-t(y-2)^2 + 8t\right)\ dy\ N$, determine the angular velocity of the gate after $t = 2\ seconds$ if the gate initially starts at rest.



FBD



$$F = \int dF = \int_{0m}^{2m} dF = \int_0^2 \left(-t\,(y-2)^2 + 8t\right) dy$$

$$= t \int_0^2 -y^2 + 4y + 4\ dy = t\left[-\frac{1}{3}y^3 + 2y^2 + 4y\right]_0^2$$

$$F = \frac{40}{3}t$$

Find a : $\int_0^a y\,dF = \int_a^2 y\,dF$

$$\cancel{t}\int_0^a y(-y^2+4y+4)\,dy = \cancel{t}\int_a^{2m} y(-y^2+4y+4)\,dy$$

$$\int_0^a -y^3+4y^2+4y\,dy = \int_a^{2m} -y^3+4y^2+4y\,dy$$

$$-\frac{1}{4}a^4+\frac{4}{3}a^3+2a = \left(-\frac{1}{4}(2)^4+\frac{4}{3}(2)^3+2(2)^2\right)$$
$$-\left(-\frac{1}{4}a^4+\frac{4}{3}a^3+2a^2\right)$$

$$-\frac{1}{2}a^4+\frac{8}{3}a^3+4a^2 = 14.\overline{66}$$

$a = 1.465\,m$ or $\cancel{6.463\,m}$

Impulse & Momentum :

$$\cancel{I_p\omega_1}^{\,\omega_1=0} + \sum\int_{t_1}^{t_2} M_p\,dt = I_p\,\omega_2$$

$$0 + \int_0^t a\,F\,dt = \frac{1}{12}mb^2\,\omega_2$$

$$\int_0^{2\,sec}(1.465\,m)\left(\frac{40}{3}t\right)dt = \frac{1}{12}(20\,kg)(2\,m)^2\,\omega_2$$

$$(1.465\,m)\left(\frac{40}{3}\right)\left[\frac{1}{2}t^2\right]_0^2 = \frac{1}{12}(20\,kg)(2\,m)^2\,\omega_2$$

$$\boxed{\omega_2 = 5.86 \text{ rad/s}}$$