Problem **20-R-IM-DK-3**

In this problem, we're asked to find the angular velocity of the gate after a time of two seconds, given the force distribution df that we're, which is given, which is a function of y, and T. Okay, we're also given, we're also told that the gate consists of a slender rod, which is 20 kilograms, on which a 20 kilogram thin plate is attached to. And we're given the dimensions of the plates, a 1.5 meters and B two meters. So the first thing we need to do is understand what is going on. So we have a force distribution over here along the direction of y, which changes with y. So changes with the distance from the top of the gate to the bottom. Hey, so you can see that this force distribution here is not even it actually changes somehow with distance. Okay. And it also varies with teeth. Okay. And this is similar This is because hydrostatic pressure changes the force from based on the depth. And also the pump changes it with time. And so we, what we need to do is we need to find the equivalent force that acts on this plate. So this is going to be called F. And it's the equivalent for us, but also the distance where this force acts, okay. So that distance there is the distance that we need to find. Okay. And we'll call that little distance, little eight. Okay. So this is because all of this, this pressure here creates a total force on the plates. But this total force on so that we can just get by integrating yet, right. But this total force here, doesn't act in any location acts in a specific location, which is the location where the moments are balanced, on the top and on the bottom. I'll talk more about that later. For now, let's just focus on trying to find this force here. And then with this force, we can use impulse and momentum to solve for omega. So that's why we need to figure out this force, because when we draw the freebody diagram of this bar over here, which I'll do over here, on this freebody diagram has force so it's pinned at the top, so I'll just draw like a circle with a P, this is going to be distance a. And over here we have our force. Okay, that acts for a specific amount of time, just in this case, it's going to be two seconds, okay. And with this, we can use impulse and momentum to solve for the Omega, the final omega, okay, so that's why we need to find that for us. Okay. Now, let's solve for F. So we know that f is going to be the integration of d f. Okay, and it's going to be the integration over the whole distance, so from y equals to zero to Y equals to two, okay, so it's going to be the integral from zero to two meters and meters of D F, which is also equal to the integral from zero to two, five, and we just plug in this equation here. So negative t, y minus two squared plus HTD y, okay? And we can take out the T, because T doesn't depend on y. And it's in both terms of the equation of the integral, and we can simply integrate and get the following is equal to T times the integral from zero Two to negative y squared plus four y plus four d y, which is also equal to t times negative 1/3 y cubed plus two y squared plus four y, integrated from zero to two, and f is going to be equal to 40 over three t. So that's the force. And again, this depends on time, because this force, but the time portion of it, it's dependent on the pump, because the pump is not, doesn't apply a constant input, because it's an old pump. Okay, so now that we have the force, we need to find the equivalent location of the force, which is little a, okay. So to find little a, we need to essentially use the following equation, which I'll explain a little bit. So find a, so we have the integral from zero to a y df being equal the integral from a two to y df. Okay. So what does this mean? So df is infinitesimal amount of force at a location. Okay? And why is the location about this point P here, so I forgot to draw in the coordinate system. But why is done this way? x is positive that way, okay. So what we're trying to essentially find is balancing the moments between zero to a. So between zero to a, is going to be equal from A to P. And these are the moments about this point P yr. Okay. So this is what this equation represents. So we have to equate the moments on each side of the equivalent force. Okay. So we can plug everything in here and solve this equation, solve the two integrals, equate them and get the following. So this will be a bit long to integrals are quite long. Okay. So we have, I'm going to already pull the T out, because just like before t came out of the integral because it's an every term of the integral. So we have T times the integral from zero to a of y times negative y squared, plus four y plus four DUI, which is equal to T times the integral from a to meters of y times negative y squared, plus four y plus four d y. Okay. And then we multiply these y's into each of these terms, and this into this, we can also directly cancel the T, because it's in both sides of the equation. And we get the following integrals, the integral from zero to a of negative y cubed plus four y squared plus four y, d y is equal to the integral from a to two meters of negative y cubed plus four y squared plus four y, d y. And then we solve these integrals and we get the following. negative one over four A to the power four plus four over a three A to the three plus two, a is equal to negative one over four times two to the four plus four over three times two to the three, plus two times two squared. minus negative one over four, a to the four, plus four over three, a to the three plus two A squared. Okay, so I plugged everything in and this side has less turns because there's this zero here, which cancels everything out and said here, there's a and two. So that's why you see the two and eight terms. Okay? Now we can simplify this to the following negative one over two, a to the four plus eight over three, a cubed plus four, a squared is equal to 14.66 repeated. Okay, so I just simplified the above equation. Okay, now we can solve for this. And this, you can solve using a solver. And you get the following values of a, so a is going to be equal to either 1.465 meters, or 6.463 meters. Now we can see that this distance is bigger than the two meter distance, how of this whole thing so that can't be an answer. So that's why we scrap this one. And we pick this one here. Okay, we pick that one because that is the only answer that fits between the zero and two meters, which are the bounds the length of that plate. Okay. So now that we've picked our we've determined this distance a and this force F, we can do the we can use impulse and momentum to solve for the final omega. Okay, so this is what the impulse and momentum equation looks like. So impulse and men term. This is what it looks like. So I p omega one plus the sum of integral from t one to T two of MP, dt is equal to IP omega two. Okay. So what we have is since IP omega one is going to be made up of two components, the rod and the plate because we're assuming that this barrier is made of a rod and a plate, we have two components, but we also know that omega one is zero, because initially it starts from rest. So we can just cancel out that term altogether, okay, so this term goes out. And this is because omega one equals to zero. Okay, the second term, we can cancel out, because we have a moment about p, which is created by that force F. That's why we found out and that's why we needed a, okay, because So this first term is going to be equal to zero plus the second term, this second term is going to be the integral from zero to t of the radius, which is a times f, okay? which is which we calculated times dt, okay. And then that's going to be equal to i, which is 112 and b squared, and then omega two, which is what we're solving for. Okay. So, this integral, we can actually compute because we have a and we have AP, we solve for both. So A is a numerical value, and f is in terms of t. Okay, so we can integrate that with respect to t and get the following. So we have the integral from zero to two, and this here is not two meters, this here is the two seconds, okay? So this is time it's not distance anymore, because here we're integrating with respect to time, but it's again, the same value but two different twos. Okay, times a which is 1.465 meters, times F, which we said was 40 over three And the time dt is going to be equal to a 112 times M, which is 20 kilograms times b squared, which is two meters squared times omega two. Okay. And here we can directly solve for omega two. Okay, so when we integrate, we get the following 1.465 meters, times 40 over three, four times one half t squared evaluated at zero and two is equal to 112 times 20 kilograms times two meters squared omega two, and when we solve for omega two, we get the following 5.86 radians per second and this is the final answer.