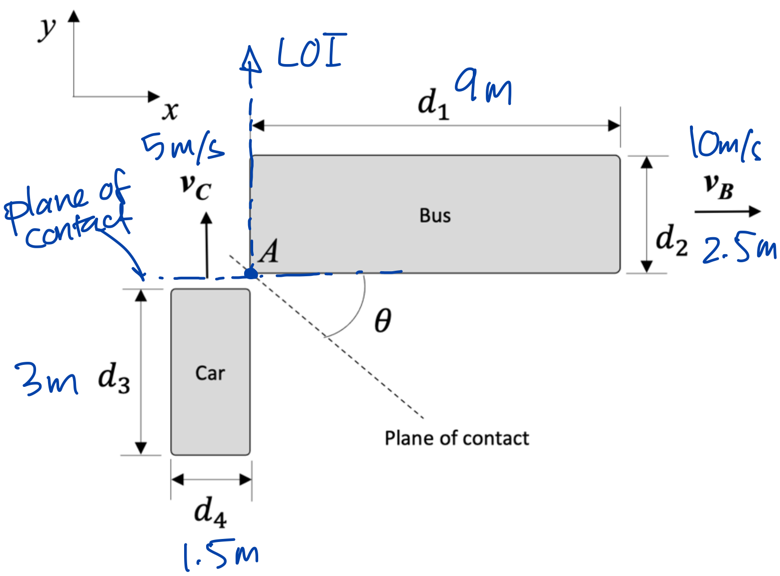
state 1

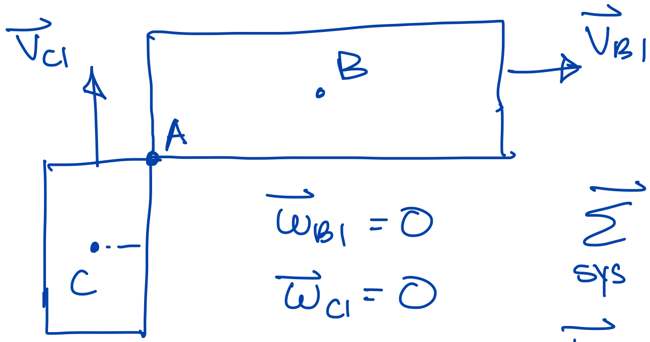
A car (m_c = 2000 kg, d_3 = 3 m, d_4 = 1.5 m) is driving on an icy road (assume frictionless road surface). It is unable to stop at an intersection and impacts a bus (m_B = 8000 kg, d_1 = 9 m, d_2 = 2.5 m). The initial vehicle velocities are $\overrightarrow{v_{c1}}$ = 5 m/s $\hat{\jmath}$ and $\overrightarrow{v_{B1}}$ = 10 m/s $\hat{\imath}$. Assume the impact is such that the vehicles corners contact at A, and the plane of contact is θ = 0°. Find the angular velocity of each vehicle just after impact, $\vec{\omega}_{C2}$ and $\vec{\omega}_{B2}$, if the impact is completely plastic at A. Assume the car and the bus can be treated as constant density objects.

state 2



Conservation of momentum (both linear and angular) for the SYSTEM (impact impulse is internal)

state 1 just before impact



$\vec{\omega}_{B1} = 0$

$\vec{\omega}_{C1} = 0$

$$\sum_{sys} \vec{J}_1 = \vec{J}_{C1} + \vec{J}_{B1}$$

$$= m_C V_{C1} \hat{\jmath} + m_B V_{B1} \hat{\imath}$$

$$\sum_{sys} \vec{K}_{A1} = \vec{K}_{A,C1} + \vec{K}_{A,B1}$$

C is car COG

$$\vec{K}_{A,C1} = I_C \cancel{\vec{\omega}_{C1}}^{0} + \vec{r}_{C/A} \times m_C \vec{V}_{C1}$$

$$= -0.75 m_C V_{C1} \hat{k}$$

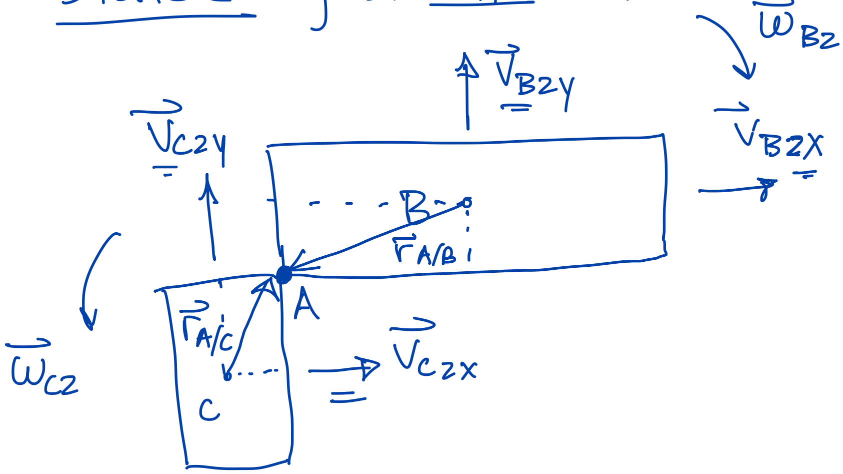
$$\vec{K}_{A,B1} = I_B \cancel{\vec{\omega}_{B1}}^{0} + \vec{r}_{B/A} \times m_B \vec{V}_{B1}$$

$$= -1.25 m_B V_{B1} \hat{k}$$

$$\sum_{sys} \vec{K}_{A1} = (-0.75 m_C V_{C1} - 1.25 m_B V_{B1}) \hat{k} = -107500 \hat{k}$$

$$\sum_{sys} \vec{J}_1 = m_B V_{B1} \hat{\imath} + m_C V_{C1} \hat{\jmath} = 80000 \hat{\imath} + 10000 \hat{\jmath}$$

state 2 just after impact



assume:

$\vec{\omega}_{B2} = -\omega_{B2}\hat{k}$

$\vec{\omega}_{C2} = \omega_{C2}\hat{k}$

$$\sum_{sys} \vec{J}_2 = (m_C V_{C2x} + m_B V_{B2x})\hat{i} + (m_C V_{C2y} + m_B V_{B2y})\hat{j}$$

$$\sum_{sys} \vec{K}_{A,2} = \vec{K}_{A,C2} + \vec{K}_{A,B2}$$

C is COG of car

$$\vec{K}_{A,C2} = I_C \vec{\omega}_{C2} + \vec{r}_{C/A} \times m_C \vec{V}_{C2}$$
$$= \left[I_C \omega_{C2} + 1.5 m_C V_{C2x} - 0.75 m_C V_{C2y}\right]\hat{k}$$

B is COG of bus

$$\vec{K}_{A,B2} = I_B \vec{\omega}_{B2} + \vec{r}_{B/A} \times m_B \vec{V}_{B2}$$
$$= \left[-I_B \omega_{B2} - 1.25 m_B V_{B2x} + 4.5 m_B V_{B2y}\right]\hat{k}$$

cons. of lin. + ang momentum $1 \to 2$:

in x: $m_B V_{B1} = m_B V_{B2x} + m_C V_{C2x} = 80000$ (1)

in y: $m_C V_{C1} = m_B V_{B2y} + m_C V_{C2y} = 10000$ (2)

about A: $K_{A1} = I_C \omega_{C2} + 1.5 m_C V_{C2x} - 0.75 m_C V_{C2y} - I_B \omega_{B2}$
$- 1.25 m_B V_{B2x} + 4.5 m_B V_{B2y} = -107500$ (3)

3 eqn's

unknowns: $V_{B2x}, V_{B2y}, V_{C2x}, V_{C2y}, \omega_{C2}, \omega_{B2}$ (6)

<u>kinematics:</u> since impact is <u>plastic</u> $\vec{V}_{A,B2} = \vec{V}_{A/C2}$

$$\vec{V}_{A,B2} = \vec{V}_{B2} + \vec{\omega}_{B2} \times \vec{r}_{A/B} \qquad \vec{r}_{A/B} = -4.5\hat{i} - 1.25\hat{j}$$

$$= V_{B2x}\hat{i} + V_{B2y}\hat{j} + 4.5\,\omega_{B2}\hat{j} - 1.25\,\omega_{B2}\hat{i}$$

$$\vec{V}_{A,C2} = \vec{V}_{C2} + \vec{\omega}_{C2} \times \vec{r}_{A/C} \qquad \vec{r}_{A/C} = 0.75\hat{i} + 1.5\hat{j}$$

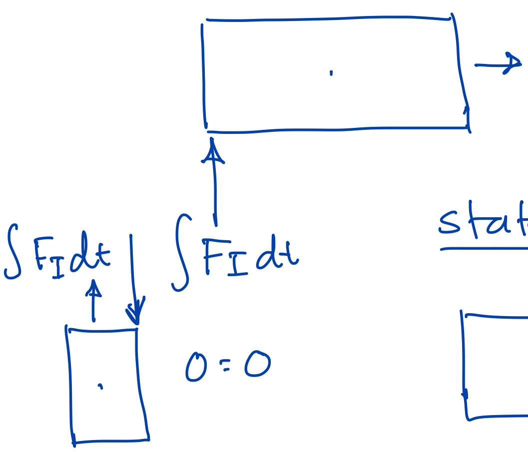
$$= V_{C2x}\hat{i} + V_{C2y}\hat{j} - 1.5\,\omega_{C2}\hat{i} + 0.75\,\omega_{C2}\hat{j}$$

$\vec{V}_A = \vec{V}_A$ (plastic, moving together)

$\hat{i}$: $V_{B2x} - 1.25\,\omega_{B2} = V_{C2x} - 1.5\,\omega_{C2}$ (4)

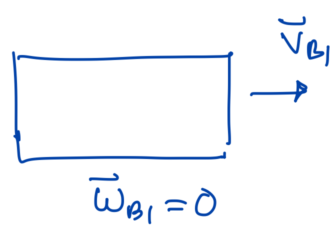
$\hat{j}$: $V_{B2y} + 4.5\,\omega_{B2} = V_{C2y} + 0.75\,\omega_{C2}$ (5)

no new unknowns (6)

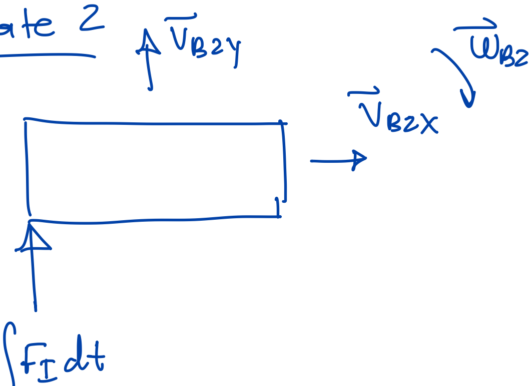


cons. of linear momentum in x ONLY for bus.

<u>state 1</u>



<u>State 2</u>



$\int \vec{F}_I dt$ in $\hat{j}$ only, lin momentum conserved in $\hat{i}$ for bus

$$V_{B1} = V_{B2x} = 10 \text{ m/s} \quad (6)$$

6 eqn
6 unknown $\Rightarrow$ solve.

① + ⑥: $80000 = m_B v_{B2x} + m_C v_{C2x}$ $\quad v_{B2x} = v_{B1} = 10\,m/s$

$$80000 = (8000)(10) + m_C v_{C2x} \Rightarrow \underline{v_{C2x} = 0}$$

②: $10000 = 8000\, v_{B2y} + 2000\, v_{C2y} \quad / 2000$

$$\Rightarrow 5 = 4 v_{B2y} + v_{C2y} \Rightarrow v_{C2y} = 5 - 4 v_{B2y}$$

③ + ① + ⑥: $-107500 = I_C \omega_{C2} + 1.5 m_C \cancelto{0}{v_{C2x}} - 0.75 m_C v_{C2y} - I_B \omega_{B2}$

$$- 1.25 m_B \cancelto{v_{B1}}{v_{B2x}} + 4.5 m_B v_{B2y}$$

$$I_C = \frac{1}{12}(2000)(3^2 + 1.5^2) = 1875\ kg\text{-}m^2$$

$$I_B = \frac{1}{12}(8000)(9^2 + 2.5^2) = 58167\ kg\text{-}m^2$$

$$\Rightarrow -107500 = 1875\,\omega_{C2} - 1500\, v_{C2y} - 58167\,\omega_{B2}$$

$$- 100000 + 36000\, v_{B2y}$$

$$\Rightarrow -7500 = 1875\,\omega_{C2} - 1500\, v_{C2y} - 58167\,\omega_{B2} + 36000\, v_{B2y}$$

④ + ① + ⑥: $\cancelto{v_{B1}}{v_{B2x}} - 1.25\,\omega_{B2} = \cancelto{0}{v_{C2x}} - 1.5\,\omega_{C2}$

$$\Rightarrow 10 - 1.25\,\omega_{B2} = -1.5\,\omega_{C2}$$

⑤: $v_{B2y} + 4.5\,\omega_{B2} = v_{C2y} + 0.75\,\omega_{C2}$

③ + ②: $-7500 = 1875\,\omega_{C2} - 1500(5 - 4 v_{B2y}) - 58167\,\omega_{B2} + 36000\, v_{B2y}$

$$\cancel{-7500 + 5(1500)} = 1875\,\omega_{C2} - 58167\,\omega_{B2} + (36000 + 4(1500))\, v_{B2y}$$

$$0 = 1875\,\omega_{C2} - 58167\,\omega_{B2} + 42000\, v_{B2y}$$

⑤ + ②: $v_{B2y} + 4.5\,\omega_{B2} = (5 - 4 v_{B2y}) + 0.75\,\omega_{C2}$

$$\Rightarrow 5 v_{B2y} = 5 - 4.5\,\omega_{B2} + 0.75\,\omega_{C2}$$

$$\rightarrow \quad V_{B2y} = 1 - \frac{4.5}{5}\omega_{B2} + \frac{0.75}{5}\omega_{C2}$$

$$⑤ \Rightarrow ③: \quad 0 = 1875\,\omega_{C2} - 58167\,\omega_{B2} + 42000\left(1 - \frac{4.5}{5}\omega_{B2} + \frac{0.75}{5}\omega_{C2}\right)$$

$$-42000 = (1875 + 6300)\,\omega_{C2} - (58167 + 37800)\,\omega_{B2}$$

$$-42000 = 8175\,\omega_{C2} - 95967\,\omega_{B2}$$

$$+\quad 5450 \times \left(10 = -1.5\,\omega_{C2} + 1.25\,\omega_{B2} \right)$$

$$54500 = -8175\,\omega_{C2} + 6812.5\,\omega_{B2}$$

$$12500 = 0 - 89154.5\,\omega_{B2}$$

$$\rightarrow \quad \omega_{B2} = -0.140 \text{ rad/s}$$

$$\omega_{C2} = \frac{1}{-1.5}\left(10 - 1.25\,\omega_{B2}\right) = -6.78 \text{ rad/s}$$

$$\boxed{\begin{aligned} \vec{\omega}_{B2} &= 0.14 \text{ rad/s } \hat{k} \\ \vec{\omega}_{C2} &= -6.78 \text{ rad/s } \hat{k} \end{aligned}}$$