Question 1:

Find an expression for the angular natural frequency of the following system, and find the maximum amplitude of vibration of the system with mass $m = 10$ kg and spring constant $k = 200$ N/m when given an initial displacement of $x_0 = 0.1$ m and an initial velocity of $v_0 = 0.3$ m/s.

**FBD perturbed position**

![FBD diagram](image)

\[ \mathbb{F}_x : -F_k - F_k = ma_x = m\ddot{x} \]

\[ F_k = kx \]

\[ \Rightarrow -2kx = m\ddot{x} \]

\[ \Rightarrow m\ddot{x} + 2kx = 0 \]

**ODE solution:** $x(t) = C \sin(w_0 t + \phi)$

\[ C = \left( \frac{v_0^2}{w_0^2} + x_0^2 \right)^{1/2} = \left( \frac{v_0^2 m}{2k} + x_0^2 \right)^{1/2} \]

\[ = \left( \frac{(0.3 \text{m/s})^2 (10 \text{kg})}{2 (200 \text{N/m})} + (0.1 \text{m})^2 \right)^{1/2} \]

\[ C = 0.1107 \text{m} \]

**Normal form:**

\[ \ddot{x} + \frac{2k}{m} x = 0 \]

\[ x + \omega_n^2 x = 0 \]

\[ \omega_n = \sqrt{\frac{2k}{m}} \]

\[ k_{eq} = 2k \]

\[ \Rightarrow \omega_n = \sqrt{\frac{k_{eq}}{m}} \]

\[ w_n = \frac{2k}{m} \]