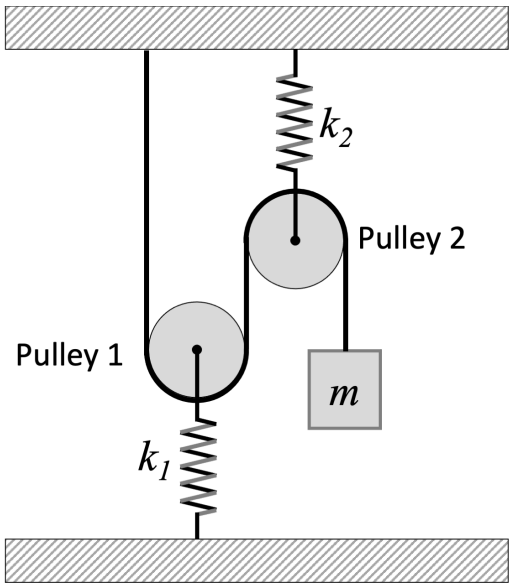
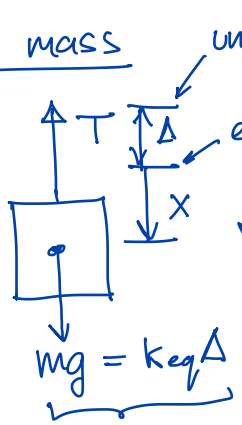
Problem 1 [10 marks] Find the natural angular frequency, ω_n, for the system shown. The pulleys are frictionless and have negligible mass. Values are: m = 8 kg, k_1= 300 N/m, k_2 = 400 N/m.



Rope has same tension (no stretch, frictionless pulleys).

FBD mass



$\Sigma F_x: -T = m\ddot{x}$

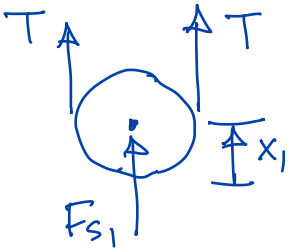
$T = k_{eq}x$

$m\ddot{x} + k_{eq}x = 0$

$\ddot{x} + \frac{k_{eq}}{m}x = 0$

ω_n^2

FBD pulley 1 (m=0)



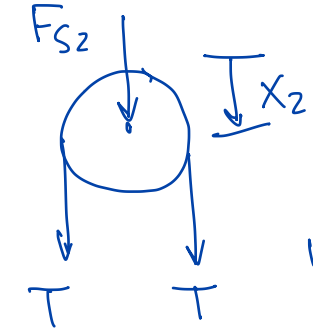
$\Sigma F_x: 2T + F_{s1} = 0$

$F_{s1} = -k_1 x_1$

$\Rightarrow 2T = k_1 x_1$

movement of rope $= 2x_1$

FBD pulley 2 (m=0)



$\Sigma F_x: 2T + F_{s2} = 0$

$F_{s2} = -k_2 x_2$

$\Rightarrow 2T = k_2 x_2$

movement of rope $= 2x_2$

$x = 2x_1 + 2x_2$

$\frac{T}{k_{eq}} = 2\left(\frac{2T}{k_1}\right) + 2\left(\frac{2T}{k_2}\right)$

$k_{eq} = \frac{1}{4}\frac{k_1 k_2}{k_1 + k_2}$

$\omega_n = \sqrt{\frac{k_{eq}}{m}}$

$= \sqrt{\frac{1}{4m}\frac{k_1 k_2}{k_1 + k_2}}$

$\omega_n = 2.31$ rad/s