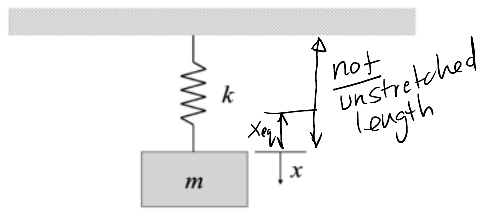
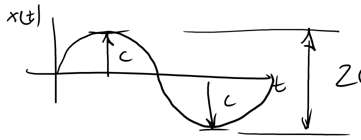
15-1 WP002

July 25, 2019 3:24 PM

Question 2:

Determine the equation of motion of the system from Newton's Second Law. Assume mass m = 5 kg and spring constant k = 500 ~~m~~ N/m. Find the initial displacement, x_0, such that the mass oscillates over a total range of 4 m. Assume the initial perturbation velocity, v_0, is 10 m/s.





$2C = 4m$

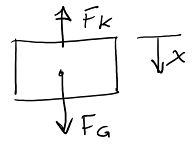
$$2C = \cancel{4}^{2}m = 2\sqrt{\frac{V_0^2}{\omega_n^2} + X_0^2} = \cancel{2}\sqrt{\frac{10^{\cancel{2}}}{\cancel{100}}^{1} + X_0^2}$$

$$2 = \sqrt{1 + X_0^2}$$

$$4 = 1 + X_0^2$$

$$3 = X_0^2 \Rightarrow \boxed{X_0 = 1.73\,m}$$

FBD equilibrium

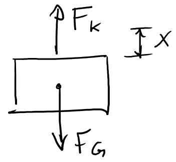


$\Sigma F_x: F_G - F_k = 0$ (static)

$mg - kX_{eq} = 0$

$mg = kX_{eq}$ ← always true

FBD perturbed



$\Sigma F_x: F_G - F_k = ma = m\ddot{x}$

$mg - k(x_{eq} + x) = m\ddot{x}$

$\underbrace{mg - \cancel{k}x_{eq}}_{=0!} - kx = m\ddot{x} \Rightarrow m\ddot{x} + kx = 0$ ← all terms positive ✓ mg not included

normal form:

$\ddot{X} + \frac{k}{m}X = 0 \Rightarrow \omega_n^2 = \frac{k}{m}$

$\Rightarrow \ddot{X} + \omega_n^2 X = 0$

$= \frac{500\,N/m}{5kg}$

$= 100\ rad/s^2$

solution:

$x(t) = C\sin(\omega_n t + \phi)$

$C =$ amplitude

$= \sqrt{\frac{V_0^2}{\omega_n^2} + X_0^2}$