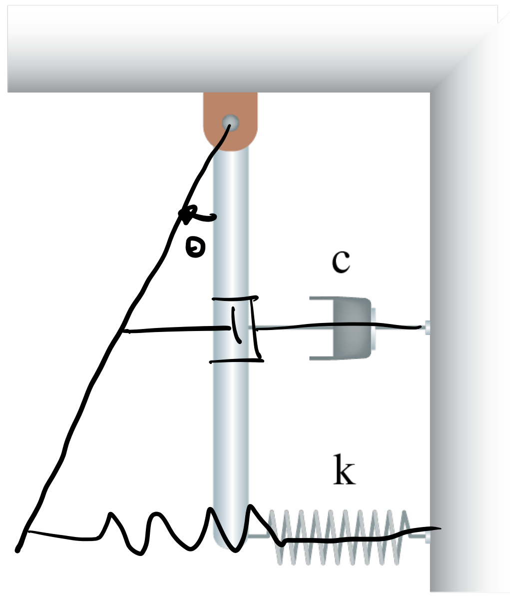
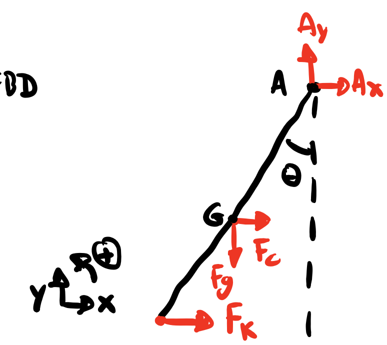
A bar of length $1.5m$ mass of $2kg$ is pinned to the ceiling. A spring, $k = 50N/m$, is attached to the bottom of the bar and a damper, $c = 10Ns/m$, is attached halfway down. Given a small angle displacement, find the damped frequency and the roots.



FBD



$$F_c = cv = c\dot{\theta}\frac{l}{2}$$

$$F_k = kx = kl\theta$$

$$\Sigma M_A = I_A \alpha$$

$$F_g = mg$$

$$F_c \frac{l}{2}\cos\theta + F_k\, l\cos\theta + F_g \sin\theta = -\frac{1}{3} m l^2 \ddot{\theta}$$

small $\theta \Rightarrow \cos\theta \simeq 1$ & $\sin\theta \simeq \theta$

$$\frac{c\ell^2}{4}\dot{\theta} + (k\ell^2 + mg)\theta + \frac{1}{3}m\ell^2\ddot{\theta} = 0 \quad \Rightarrow \text{EOM}$$

$$m'\ddot{\theta} + c'\dot{\theta} + k\theta = 0$$

$$c' = \frac{c\ell^2}{4} = \left(10\ \frac{Ns}{m}\right)\frac{(1.5\ m)^2}{4} = 5.625$$

$$k' = k\ell^2 + mg = \left(50\frac{N}{m}\right)(1.5\ m)^2 + (2\ kg)(9.81\ m/s^2) = 132.12$$

$$m' = \frac{1}{3}m\ell^2 = \frac{1}{3}(2\ kg)(1.5\ m)^2 = 1.5$$

$$\omega_n = \sqrt{\frac{k'}{m'}} = \sqrt{\frac{132.12}{1.5}} = 9.385\ \frac{rad}{s}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\zeta = \frac{c'}{2\sqrt{m'k'}} = \frac{5.625}{2\sqrt{(1.5)(132.12)}} = 0.2$$

$$\omega_d = \left(9.385\ \frac{rad}{s}\right)\sqrt{1-0.2^2} = 9.196\ \frac{rad}{s}$$

$$\boxed{\omega_d = 9.196\ \frac{rad}{s}}$$

$$r_{1,2} = \frac{-c'}{m'} \pm i\,\frac{\sqrt{4m'k' - c'^2}}{2m'} = \boxed{-3.75 \pm i\,9.196}$$