Problem **20-R-VIB-DY-26**

In this problem, there is mass attached with a spring to a wall. And this mass has friction between it and bottom surface. And we're asked to determine how long it takes for this mass to come to a complete stop if we apply an initial displacement of one meter from the equilibrium position. So the first thing we need to do is like always draw a freebody diagram and of the block and apply all the forces. So we have the block, and we have x in this direction. So we have the force of friction pulling back and the forest user the spring pulling back, or call this f k. And then we have normal force, and we have mg. Okay, so this is a freebody diagram, and we can get the equation of motion by doing a sum of forces in the x and in the y direction. So in the x direction, we have f of f plus f k is going to be our minus their voting the negative x direction, negative A minus, okay, x double dot. And so this, we get x double dot plus mu g plus k x is equal to zero. This is essentially plugging in UMG into the force of friction and k x into f k. Okay, and this m g comes from n, which is mg, because if we do a sum of forces in the Y, that's what we get, okay. So, when we solve this equation here, the solution that we get is going to be in this in the following form x of t is equal to x naught minus two n minus one mu g over k times cosine of omega and t plus mu g over k times negative one to the n plus one. Okay. And this is n represents every peak. Okay. So, essentially what this equation looks like, Is this all dried out here. So, it starts over here, and then it sinusoidally decreases like that, okay, so this is the first peak, we have a second peak, we have a third and we have a fourth and we have a fifth, okay. And essentially at the peak is when we get a change in direction. So, when the mass instantly stops and then starts moving backwards, okay? So, what we need to determine is how many peaks are required for this system to stop. So, you displace by one meter, so, you add an X of one meter, and then this thing is going to start oscillating back and forth, back and forth, back and forth. And we need to determine when it stops oscillating, so, we know that at the point at each of these peaks here, what happens is a change of direction so, the mass has to temporarily stop and that is where we have static friction, okay. So, once we get static friction that is higher, so the force due to the static friction is higher then the spring force, then what happens is the block doesn't move anymore because friction because the force of the spring is overcome by the friction, which is static friction, so it's stationary. Okay, so What we need to determine is how many of these peaks, so how many ends are required for this whole system to stop. And that is based on the force of friction. Okay, so we have the following statement F, F to be bigger than f, k, okay. And so we have to stop at a peak, because at this peak is where we have this static friction. And so what we need to do is plug in the values for these. So the force of friction, f of f is just new mg. And it has to be bigger than f k, fk is just k x, right? So k times this x term here, so, okay times X of t, and I take the absolute value because sometimes this can be a negative force depending on the direction. Honestly, if you take the, we just care about the absolute value, not the actual value. Okay, so we can see from this solution here, so this, I just saw this differential equation, I'm not going to solve it here. But if you just go online, you can solve it, and you will get this this form, we have x naught, which is the initial displacement, then we have this term over here, which is almost played by cosine of omega t. And then we have this new term here and we have this exponent here, because of the directionality of friction, okay, so, we have this equation, we can plug in this X of t into here, and we can rearrange and we get the following. So, mu g over k has to be bigger than the absolute value of x naught minus mu g over k times two n minus one of cosine omega n t plus this is new g over k, negative one to the n plus one. Okay. And again absolute value. Alright, so now that we have this we can try, we need to get rid of this absolute value and simplify this equation to solve for n. Okay, so, what we need to do is we first are going to plug in some values to get numbers instead of all of these constants. So, mu m g over k, we can simplify that to 0.04905. Okay, and that's going to have to be bigger than x naught, we can simplify the above equation to the following and I'll explain how to do it minus 0.049052 n minus two. Okay, so we can simplify the following this equation here into this one. So, first thing we do is wherever there's mu mg over k, we plug in the values and we get the following here in here. Okay, now, we can get with this here is minus one, right, but if we get rid of this absolute value, this here is always going to be positive. So that's why we're going to subtract since this is negative, here, we're going to subtract another one. So that's why we get negative minus two. Okay, so we included this term with this because of the absolute value. And then this x naught can just be pulled out of the absolute value. And this cosine of omega n t here, we are going to pull that out because that's just going to be equal to one. Okay, because we're looking at a peak, the cosine at a peak is always one. So that's why this becomes just becomes one. So it should be multiplied into this term here, like to this whole term here, but this becomes one so that's why we can also include this into there. Okay. So we get the following. and here we can solve for and okay. Because we're given an x naught, and we can solve for n, so here we have. And so we've x naught is equal to one meter, we got that n is equal to 11. Okay, so we need 11 ends, but one n is half a cycle. So that's why this will be five full cycles. Right that better. We need 11 ends, but that's five full periods or five full cycles. And therefore, once to find out the time, we can plug everything into this cosine here, which we can plug everything here which this cosine needs to be equal to one. So this omega n t here has to be equal to 10 times pi, right so that this whole cosine term becomes one. So we have omega and t is equal to 10 pi because that is five periods 10 pi. And that will yield this cosine to be one. So we need to find omega n and omega n is simply root k over m. So square root of k over m, which is equal to the square root of 40 radians per second. And so we can get t is equal to 10 pi over square root of 40 seconds. And that is the final answer for this problem.