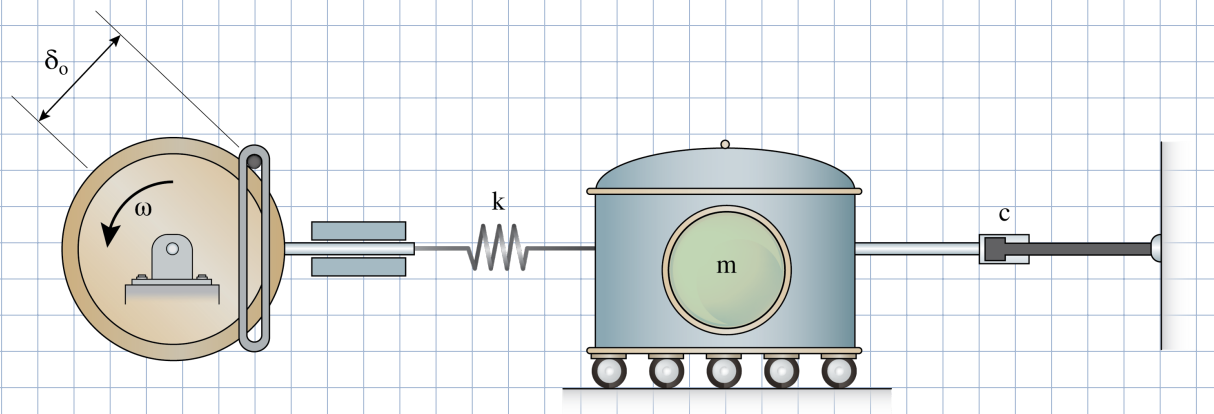
Your latest invention is a milkshake maker that uses vibrational movement to create the perfect milkshake. You start by adding all the frozen ingredients to the milkshake maker and you can approximate it as a uniform, solid container. The milk shaker and all the ingredients inside have combined mass of m = 5.2 kg. It is connected to a damper with damping constant c = 8 N·s/m on one side, and a spring of stiffness k = 39 N/m on the other. A rotating wheel causes periodic motion to keep the milkshake shaking where δ_0 = 41 cm and the angular velocity is ω = 4 rad/s.

Find the damping ratio ζ, the phase angle φ′ of the steady state solution, the natural period of oscillation τ_n, the period of the steady state response τ_0, and the period of the damped vibration τ_d.



Damping ratio: $\zeta = \frac{c}{c_c} = \frac{c}{2 m \omega_n} = \frac{8\ N\cdot s/m}{2(5.2\ kg)(2.739\ rad/s)} = 0.2808 = \zeta$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{39\ N/m}{5.2\ kg}} = 2.739\ rad/s$

Phase: $\phi = \tan^{-1}\left[\frac{2\zeta(\omega_0/\omega_n)}{1-(\omega_0/\omega_n)^2}\right] = \tan^{-1}\left[\frac{2(0.2808)(4/2.739)}{1-(4/2.739)^2}\right]$

$\phi = -0.627\ rad$

Periods: $\tau_n = \frac{2\pi}{\omega_n} = 2.294\ sec$

$$\tau_o = \frac{2\pi}{\omega_o} = \boxed{1.571 \text{ sec}}$$

$$\tau_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{2.629 \text{ rad/s}} = \boxed{2.390 \text{ sec}}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 2.629 \frac{\text{rad}}{\text{s}}$$