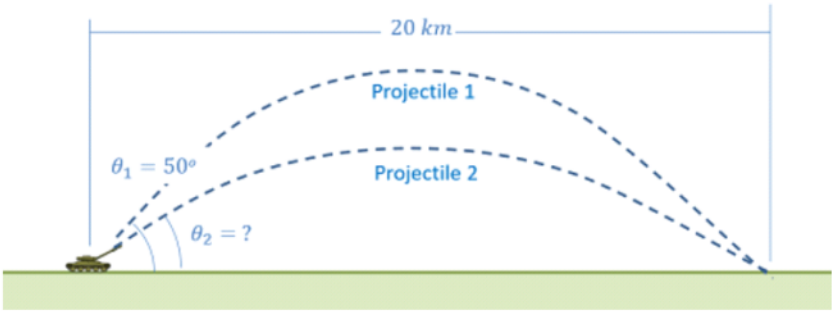
Question 4

You are designing an artillery system with the aim of hitting a target 20 km away with two shells at the same time.

- The first shot will be fired at a 50-degree angle. What is the required initial velocity needed to hit the target? Assume air resistance is negligible.
- The second shot will be fired five seconds after the first. What is the required initial velocity and angle to hit the target at the same time as the first shot?



Projectile 1

$\ddot{x}(t) = 0$ $\qquad$ $\ddot{y}(t) = -9.81$

$\dot{x}(t) = V_0 \cos(50)$ $\qquad$ $\dot{y}(t) = -9.81t + V_0 \sin(50)$

$x(t) = V_0 \cos(50)t$ $\qquad$ $y(t) = -\frac{9.81}{2}t^2 + V_0 \sin(50)t$

V_0 to hit target

$x = 20{,}000 = V_0 \cos(50)t$ $\qquad$ $y = 0 = -\frac{9.81}{2}t^2 + V_0 \sin(50)t$

$$\boxed{V_0 = 446.35 \text{ m/s}}$$

$$t = 69.709 \text{ s}$$

Projectile 2

$$t = 69.709 - 5 = 64.709_s$$

$$\ddot{x}(t) = 0 \qquad \ddot{y}(t) = -9.81$$

$$\dot{x}(t) = V_0 \cos(\theta) \qquad \dot{y}(t) = -9.81t + V_0 \sin(\theta)$$

$$x(t) = V_0 \cos(\theta) t \qquad y(t) = \frac{-9.81}{2} t^2 + V_0 \sin(\theta) t$$

V_0 + θ to hit target

$t = 64.709$

$$x = 20{,}000 = V_0 \cos(\theta)(64.709) \qquad y = 0 = \frac{-9.81}{2}(64.709)^2 + V_0 \sin(\theta)(64.709)$$

$V_0 = 443.02$ m/s

$\theta = 45.76°$