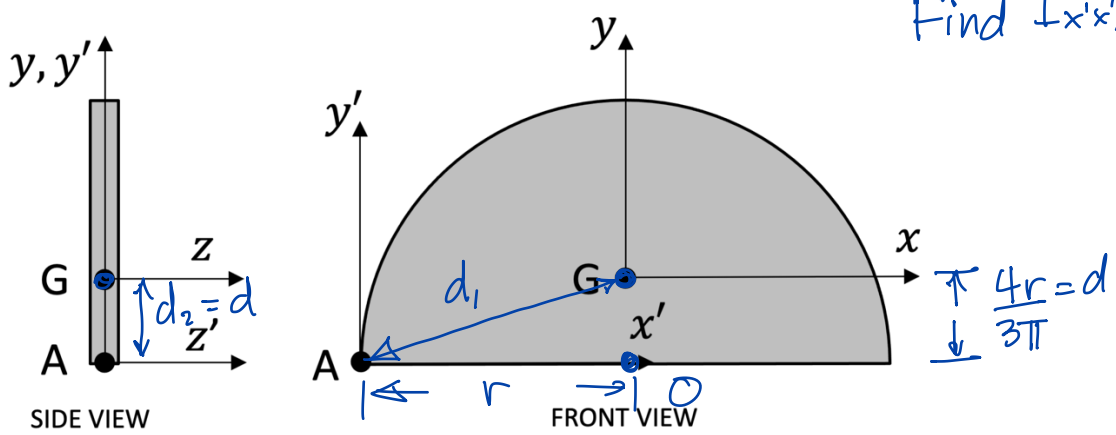
A semicircular thin plate has constant density, a radius of 10 cm, and a mass of 400 g. Find the mass moment of inertia of the plate around the axes (a) x′ and (b) z′ passing through point A.

Find $I_{x'x',A}$ & $I_{z'z',A}$



From previous problem

$$I_{zz,G} = \frac{1}{2}mr^2 - m\left(\frac{4r}{3\pi}\right)^2$$

$$I_{xx,G} = \frac{1}{4}mr^2 - m\left(\frac{4r}{3\pi}\right)^2$$

Parallel axes:

$$I_{z'z',A} = I_{zz,G} + m d_1^2 \qquad d_1^2 = \left(\frac{4r}{3\pi}\right)^2 + r^2$$

$$= \frac{1}{2}mr^2 - m\left(\frac{4r}{3\pi}\right)^2 + m\left(\frac{4r}{3\pi}\right)^2 + mr^2$$

$$= \frac{3}{2}mr^2 = \frac{3}{2}(0.4\,\text{kg})(0.1\,\text{m})^2$$

$$\boxed{I_{z'z',A} = 0.006\ \text{kg-m}^2}$$

$$I_{x'x',A} = I_{xx,G} + m d_2^2 \qquad d_2^2 = \left(\frac{4r}{3\pi}\right)^2$$

$$= \frac{1}{4}mr^2 - m\left(\frac{4r}{3\pi}\right)^2 + m\left(\frac{4r}{3\pi}\right)^2$$

$I_{xx,O}$ →

$$= \frac{1}{4}mr^2 = \frac{1}{4}(0.4\,\text{kg})(0.1\,\text{m})^2$$

$$\boxed{I_{x'x',A} = 0.001\ \text{kg-m}^2}$$